格子QCDによる $|V_{ub}|$, $|V_{ts}|/|V_{td}|$の決定

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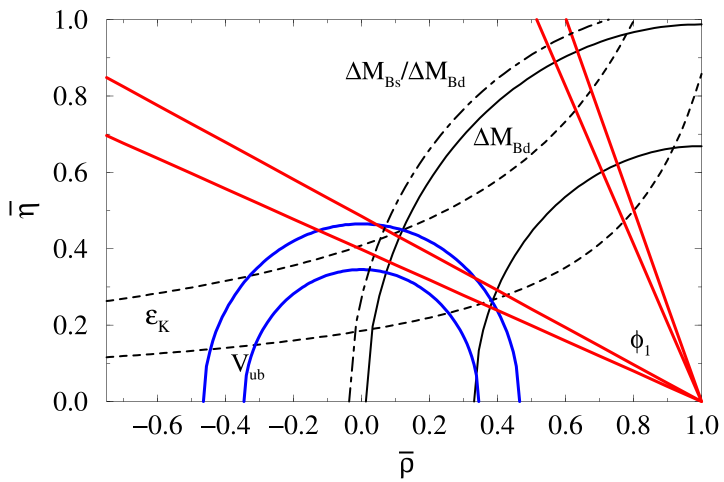
- JLQCD collaboration との共同研究:
  Nucl. Phys. Proc. Suppl. **119**, 610 (2003),
  Phys. Rev. Lett. **91**, 212001 (2003),
  Phys. Rev. D **64**, 114505 (2001),
- 福永優（広大）との共同研究:
  Phys. Rev. D **71**, 034506 (2005).

Introduction

Over-constraining the CKM elements through independent processes can probe possible signals of new physics beyond the standard model.



Role of $|V_{ub}|$ and $|V_{ts}/V_{td}|$

$|V_{ub}|$: Consistency check with ϕ_1 measurement.

$|V_{td}|$: FCNC contribution from new physics.

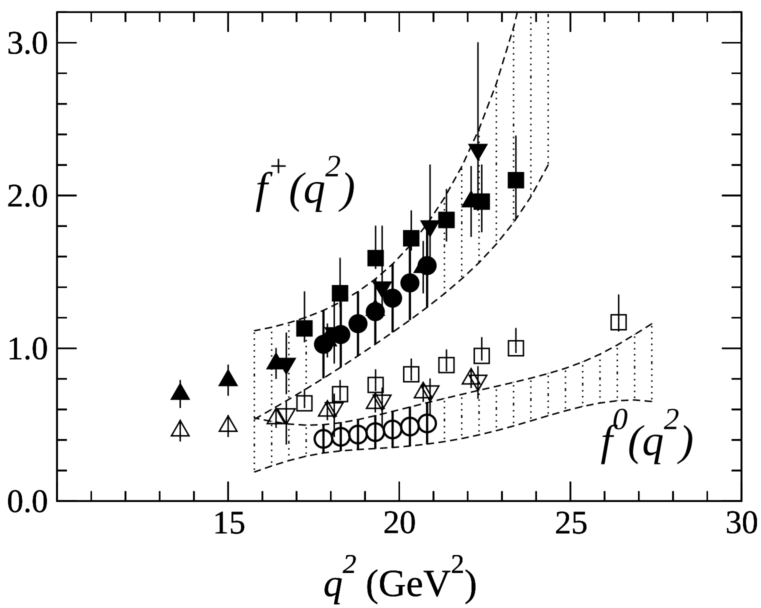
Consistency check with ϕ_3 measurement.

Determination of $|V_{ub}|$ from $B \to \pi l \nu$

$$\langle \pi(k) | \bar{q}\gamma^{\mu} b | B(p) \rangle = f^{+}(q^2) \left[(p+k)^{\mu} - \frac{m_B^2 - m_\pi^2}{q^2} q^{\mu} \right] + f^{0}(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^{\mu}, \quad (1)$$

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 [(v \cdot k)^2 - m_\pi^2]^{3/2} |f^{+}(q^2)|^2. \quad (2)$$

- Lattice cacuclation is possible only for high $q^2 (> 16\mathrm{GeV}^2)$ region.
- Experimental data is phase space suppressed for high q^2 region.
- Statistical error is larger for form factors than simpler matrix elements.
- Chiral extrapolation error also exists.



q^2 dependence of $B \to \pi l \nu$ form factors
$f^+(q^2)$ (filled) and $f^0(q^2)$ (open). $\triangle$ UKQCD, $\bigtriangledown$: APE, $\square$: Fermilab, $\bigcirc$:JLQCD.

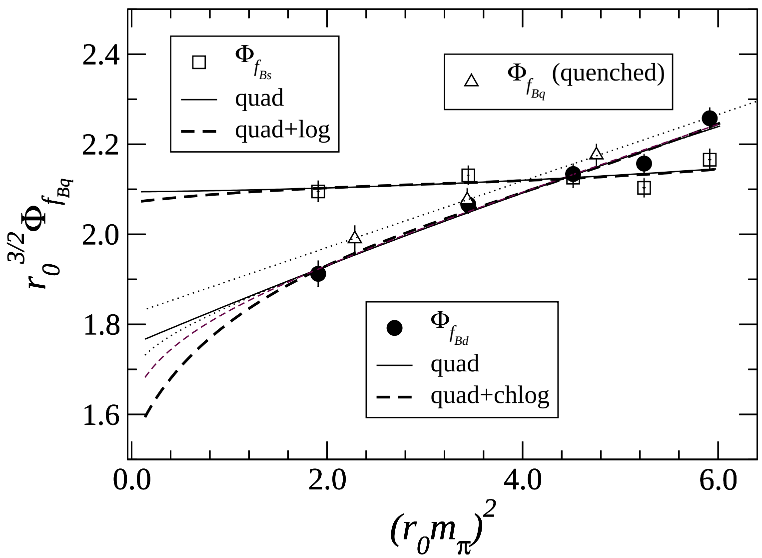
$$\pi l \nu \quad : \quad |V_{ub}| = (2.88 \pm 0.63^{+0.48}_{-0.39}) \times 10^{-3}.$$

Determination of $|V_{td}|$ and $|V_{ts}|$ from $B\overline{B}$ mixing

$$\Delta M_{B_{d(s)}} \quad \propto \quad |V_{td(s)}|^2 f^2_{B_{d(s)}} B_{B_{d(s)}}$$

- $\Delta M_{B_{d(s)}}$ will be measured at Belle, Babar, the TeVatron with 2% accuracy.
- Precise lattice determination of f_B, B_B , $\xi \equiv \frac{f_{B_s} B^{1/2}_{B_s}}{f_{B_d} B^{1/2}_{B_d}}$ are indispensable.
- Large chiral extrapolation error $\cdots$ chiral log $\sim$ 10%
  C.f. N. Yamada's talk at lattice 2001.
  A. S. Kronfeld and S. M. Ryan Phys. Lett. B 543, 59 (2002).
  S. Aoki et al. hep-lat/0307039 .

Unquenched lattice results on the decay constants $\Phi \equiv f_{HL}\sqrt{M_{HL}}$.



Chiral extrapolation of $\Phi_{f_{B_d}}$ (filled circles) and $\Phi_{f_{B_s}}$ (open squares). solid lines: polynomial fit dotted lines: fit with chiral logs

$f_{B_d} = 191(10)(^{+12}_{-22})$ MeV, $f_{B_s} = 215(9)(^{+14}_{-13})$ MeV, and $f_{B_s}/f_{B_d} = 1.13(3)(^{+13}_{-\ 2})$, $f_{B_d}\sqrt{\hat{B}_{B_d}} = 215(11)(^{+15}_{-27})$ MeV, $f_{B_s}\sqrt{\hat{B}_{B_s}} = 245(10)(^{+19}_{-17})$ MeV, and $\xi = 1.14(3)(^{+13}_{-\ 2})$.

Goal

Propose a method to improve the accuracy of
$|V_{ub}|$ and $|V_{td}/|V_{ts}|$ determination
using a feasible lattice QCD calculation .

- $|V_{ub}|$: a new method for the dispersive bound.
- $|V_{td}|/|V_{ts}|$: lattice estimate of the Grinstein ratio

$|V_{ub}|$: the dispersive bound.

Basic Idea

The differential decay rate is written as

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2}{24\pi^3}|V_{ub}|^2[(v \cdot k)^2 - m_\pi^2]^{3/2}|f^+(q^2)|^2. \tag{3}$$

In order to detemine $|V_{ub}|$ using data for all q^2 region, we need to have

- $\Gamma_i = \int_{q_i^2}^{q_{i+1}^2} dq^2 \frac{d\Gamma}{dq^2}$ from experiment $(i = 1, \cdots, N_{bin})$,
- lattice results $f^+(q_J^2)$ $(J = 1, \cdots, L)$ for large q^2,
- a reliable method to extrapolate the form factor $f^+(q^2)$ to all q^2 region,

We exploit the dispersive bound for extrapolation.

Review of the dispersive bound

An exact bound on the form factors at $f^0(q^2)$, $f^+(q^2)$ for arbitrary q^2, which can be derived from OPE and dispersion relations for the 2pt functions of vector current V^μ $(\bar{b}u)$. If we know the form factor values $f^0(q)$, $f^+(q)$ for several points $q^2 = q_J^2$ $(J = 1, \cdots, L)$, we can further restrict the bound as

$$F^{lo}(q^2; \vec{f}) \quad \leq f(t) \leq \quad F^{up}(q^2; \vec{f}) \tag{4}$$

where $F^{up/lo}(q^2, \vec{f})$ are solutions of quadratic equations whose coefficents are are known functions of

(1)	kinematical paremeters :	$q^2.m_B$,m_B^*,m_π,
(2)	OPE parameters:	Wilson coefficients and vacuum condensates.
(3)	nonperturbative inputs:inputs	$\{q_J^2\}$, $f^{0,+}(q_J^2)$ $(J = 1, \cdots, L)$.

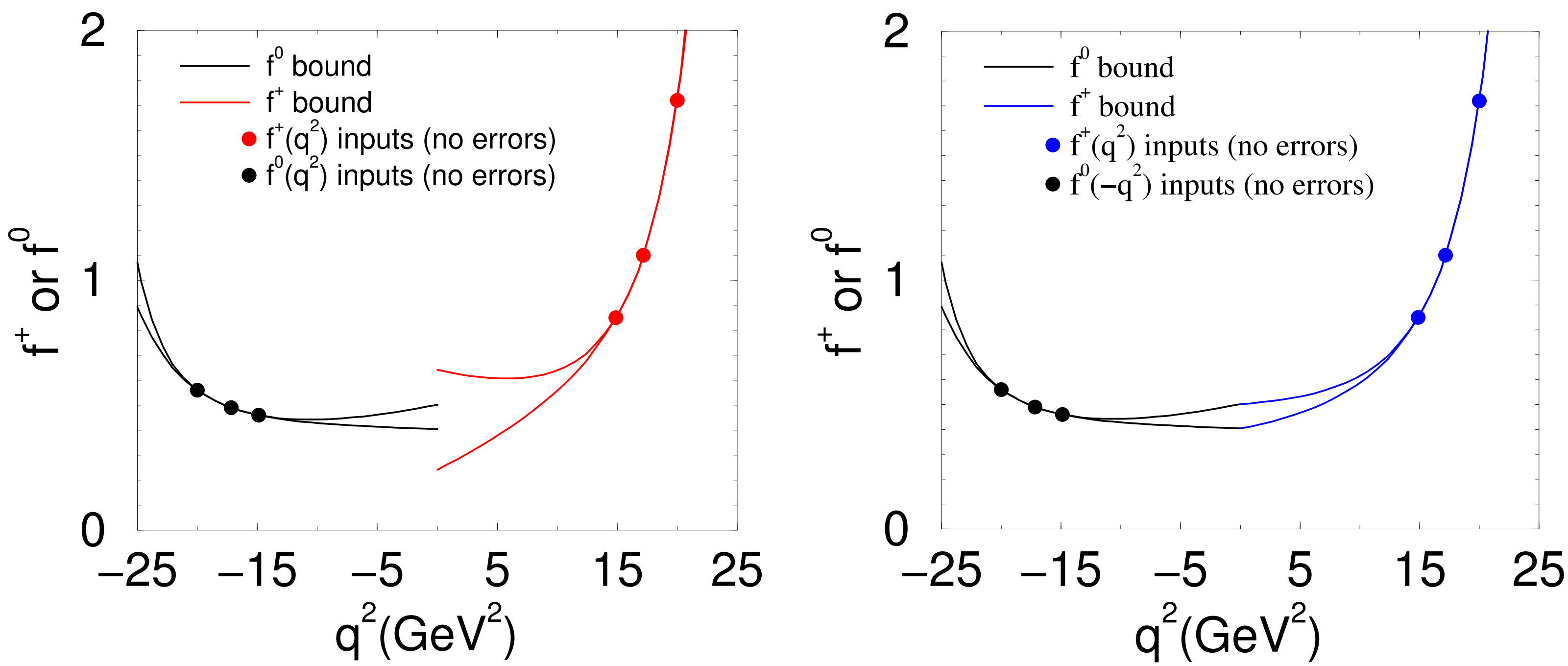


Figure 1: bound of $f^+(t)$, $f^0(-t)$ for one example set of inputs $\vec{f}$ without and with the kinematical constraint $f^+(0) = f^0(0)$

Strong model independent bound if lattice inputs have no errors.

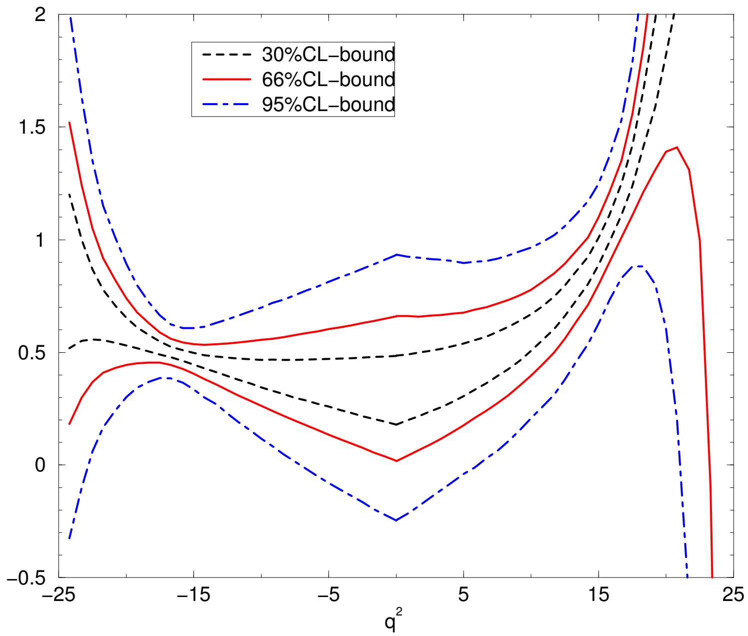
Lattice inputs $f(q_1^2), \cdots f(q_L^2)$ have errors.

$\Rightarrow$ Statistical treatment, *L. Lellouch, Nucl. Phys. B* **479**, *353 (1996)*

- Probability distribution $\mathcal{P}_{initial}$ of the dispersive bound is obtained from a random Gaussian samples based on lattice results (and errors).
- The following consistency conditions are imposed, which makes it a conditional distribution $\mathcal{P}_{condA}$.

Condition A

- The quadratic equations to determine the upper/lower bounds should have real solutions.
- The solutions of the upper/lower bounds should allow the kinematical condition $f^+(0) = f^0(0)$.



Model independent bounds on $f^{0,+}(q^2)$ at 90%,66%, 30% confidence levels.

Use of global q^2 dependence

We use the following physical condition with experimental data to reweight the probability distribution to make a new conditional distribution $\mathcal{P}_{condA+B}$.

Condition B
The experimental data Γ_i^{exp}'s should lie within
the upper and lower bounds from the theory simultaneously for all i , i.e.

$$|V_{ub}|^2\gamma_i^{lo} < \Gamma_i^{exp} < |V_{ub}|^2\gamma_i^{up} \text{ (i=1, } \cdots, N_{bin}\text{)},$$

where $\gamma_i^{up/lo}(\vec{f}) \equiv \frac{G^2}{192\pi^3 m_B^3} \int_{q_i^2}^{q_{i+1}^2} dt \left| F^+_{up/lo}(t;\vec{f}^+,\vec{f}^0)\right|^2 \lambda(q^2)^{3/2},$

Setup

Lattice form factors
(quenched JLQCD lattice data+soft pion theorem),
S. Aoki *et al.*, Phys. Rev.D 64 (2001) 114505.

q^2	$f^+(q^2)$	$f^0(q^2)$
17.79	1.03 $\pm$ 0.22	0.407 $\pm$ 0.092
19.30	1.24 $\pm$ 0.21	0.45 $\pm$ 0.11
20.82	1.54 $\pm$ 0.27	0.51 $\pm$ 0.14
q^2_{max}	$\sim \frac{f_B}{f_\pi}\frac{\hat{g}_b}{1-q^2/m^2_{B*}}$	f_B/f_π

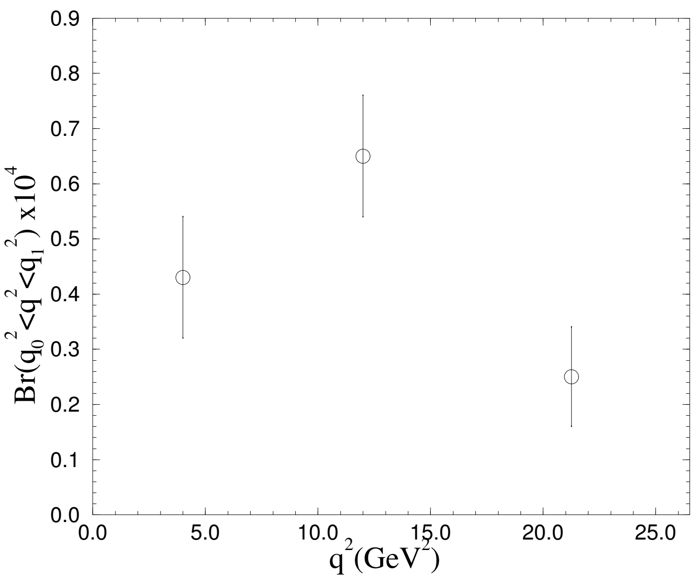
$f_B = 190 \pm 30$ MeV, (Gaussian)
$f_{B*} = 190 \pm 30$ MeV (Gaussian)
$g = [0.3, 0.9]$ (uniform). $O(10^7)$ samples are created.

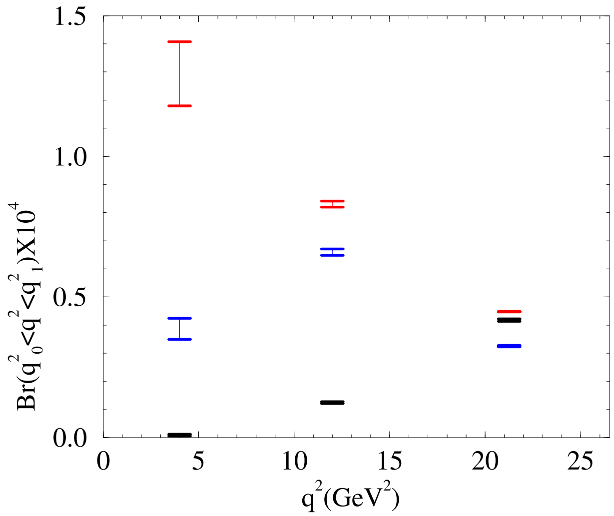
Experimental branching fraction (CLEO data),
S. B.Anthar *et al.*, Phys. Rev.D 68 (2003) 072003.

$B(0 < q^2 < 8\text{GeV}^2)$	0.43 $\pm$ 0.11
$B(8 < q^2 < 16\text{GeV}^2)$	0.65 $\pm$ 0.11
$B(16 < q^2 < q^2_{max}\text{GeV}^2)$	0.25 $\pm$ 0.09

2000 samples for $\Gamma_i \equiv \Gamma_{tot} B_i$.(Gaussian)
$|V_{ub}|$: 2000 samples for $|V_{ub}|$ which uniformly distributes over $[1.0, 6.0] \times 10^{-3}$.

Results





Branching fraction $B_i \equiv \int_{q_i^2}^{q_{i+1}^2} \frac{\frac{d\Gamma}{dq^2}}{\Gamma_{total}}$ by CLEO experiment for $q_0^2, q_1^2, q_2^2, q_3^2 =$ 0, 8, 16 GeV^2, q_{max}^2.

Upper and ower bounds of $\int_{q_i^2}^{q_{i+1}^2} \frac{d\Gamma}{dq^2}$ obtained by the dispersive bound from three different sample lattice inputs using JLQCD data. (red and black are highly unlikely statistically.)

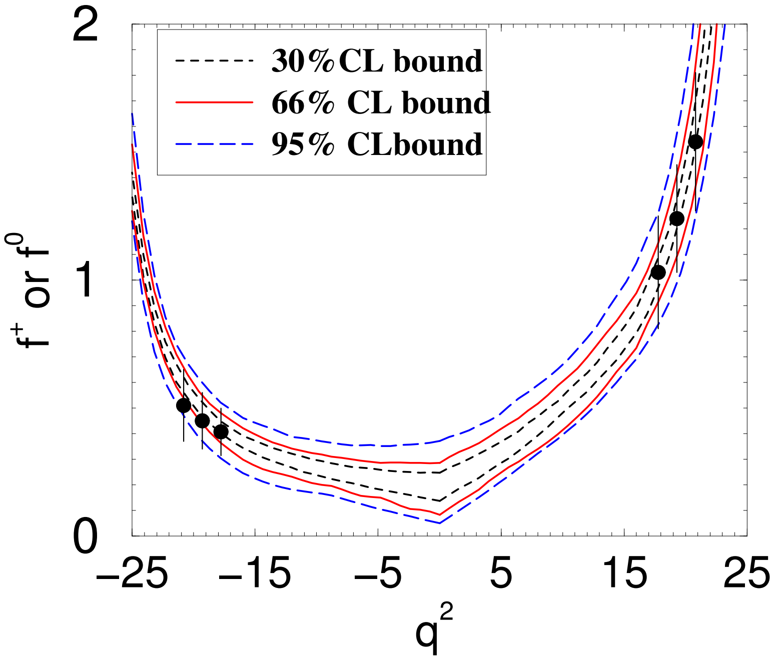


Figure 2: CLB for $f^{0,+}(q^2)$ with JLQCD's lattice input, and CLEO's experimental data

Results with JLQCD inputs

- $|V_{ub}| = (3.73 \pm 0.53) \times 10^{-3}$
- $0.126 < f(0) < 0.293$ (66% CL)

Discussions

Comparison with analysis without dispersive bounds.
JLQCD, CLEO and PDG:

$$
\begin{aligned}
\int_{q^2=16GeV^2}^{q^2_{max}} dq^2 \frac{d\Gamma(B\to\pi l\nu)}{dq^2}|_{JLQCD} &= |V_{ub}|^2(1.71\pm 0.61\pm 0.12\pm 0.44)(\text{psec}^{-1}) \\
B(q^2>16GeV^2)_{CLEO} &= 0.25\pm 0.09 \\
\tau_{B0}|_{PDG} &= 1.546\pm 0.029(\text{psec})
\end{aligned}
$$

$\Rightarrow |V_{ub}| = (3.08\pm 0.88)\times 10^{-3}$. (c.f. $|V_{ub}| = (3.73\pm 0.53)\times 10^{-3}$ from dispersive bound.)

- Errors are smaller with dispersive bound.
- Central value is changed.

$|V_{td}|/|V_{ts}|$: Grinstein ratio

Chiral behavior of f_B, f_D

Partially quenched chiral perturbation theory (*Sharpe and Zhang Phys. Rev. D53 (1996) 5125.*)

$$(\Phi) = \kappa[1 + (\Delta f_{\mathrm{Qq}}) + C_1 m_{\mathrm{q}} + \cdots]$$

where

$$(\Delta f_{\mathrm{Qq}}) = -\frac{(1+3g^2)}{(4\pi f)^2}[\frac{3}{4}m_{\mathrm{qq}}^2 \ln(\frac{m_{\mathrm{qq}}^2}{\Lambda^2})]$$

f_{B_d} : Chiral extrapolation with log term is necessary.
g is the $B^*B\pi$ coupling, where recent CLEO experiments suggests $g = 0.59 \pm 0.01 \pm 0.07$.
A. Anastassov et al. (CLEO collab.), Phys. Rev. D67 (2003) 032003.

Grinstein ratio

Ginstein considered the ratio R_1 which is close to unity.

$$R_1 = \frac{f_{B_s}}{f_{B_d}} / \frac{f_{D_s}}{f_{D_d}}.$$

(C. G. Boyd, B. Grinstein, Nucl. Phys. B 442 (1995) 205.)

In this combination, systematic errors in the chiral extrapolation are expected to cancel in the B/D ratio partially up to corrections proportional to $(m_s - m_d)/(1/M_D - 1/M_B)$.
Other systematic errors are expected to cancel in the SU(3) ratio.

Estimate of $R_1 - 1$ from chiral perturbation theory

M.Booth, hep-ph/9412228
(C.f. S.Sharpe and Y. Zhang, PRG53(1996)5125, M.Booth, PRD51(1995)2338)
In $N_f = 2$ case, numerically,

$$\begin{aligned} R_1 - 1 &= 0.17g^2 - 0.15GeV^{-1}g(g_1 - g_2) \\ &= 0.028 \sim 0.087 \end{aligned}$$

for a range of values of parameters, where g is $B^*B\pi$ coupling and g_1, g_2 are coeffs. of 1/M correction to $B^*B\pi$ coupling. Consevative estimate of error of R_1 is 5%, however,

- Some of the correction terms are neglected in the calculations.
- The true values of coefficients are not known.

Therefore, we do not know how to reduce the theoretical errors of chiral perturbation theory for R_1 any further.

Lattice Setup

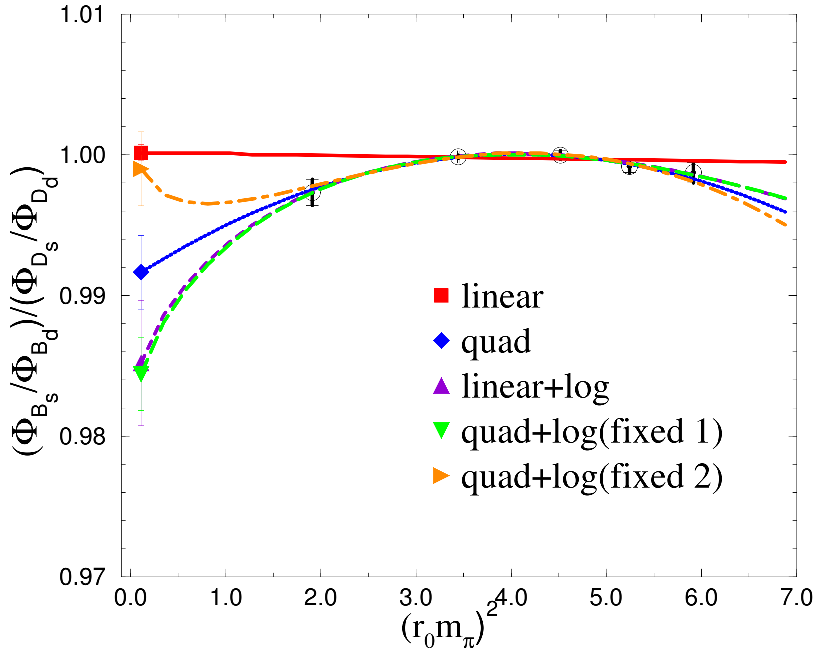
Heavy quark	: clover (Fermilab formalism)
Light quark	: clover
Gauge	: standard Wilson ($n_f = 2$ unquenched calculation)

- Both the B and D meson can be covered.
- B/D ratio can be taken.

Parameters for the simulation

$n_f = 2$, $20^3 \times 48$ lattices at $\beta = 5.2$, $c_{sw} = 2.02$. 5 light sea quark masses (κ_l).
8 heavy quark masses (κ_h) correspondings to each sea quark masses.

Our unquenched lattice results for the Grinstein ratio



Sea quark mass dependence of the Grinstein ratio $(\Phi_{B_s}/\Phi_{B_d})/(\Phi_{D_s}/\Phi_{D_d})$ in r_0 unit. The chiral extrapolation error is suppressed to 2% level.

Systematic errors

1. Chiral extrapolation
   Estimated by comparing different fitting functions. $\sim$ 0.4% (c.f. previous graph)
2. Discretization errors
   Leading error of $O((ap)^2)$ cancells in the ratio.
   Remaing dominant error is $O((ap)^2(m_s - m_d)(1/m_c - 1/m_b))$, which gives 0.6% by naive order estimation.
3. Perturbative errors
   Leading error of $O(\alpha^2)$ cancells in the ratio.
   Remaing dominant error is $O(\alpha a m_s)$ which gives 0.7% by naive order estimation.
4. Uncertainty in the strange quark mass
   Estimated by comparing result of κ_{s_2} with that of κ_{s_1}, which gives negligible errors $\sim$ 0.2%

Results

JLQCD preliminary results for the Grinstein ratio $R_1 \equiv \frac{f_{B_s}}{f_{B_d}} / \frac{f_{D_s}}{f_{D_d}}$

$$R_1 = 1.02 \quad \pm 0.01 \quad \pm 0.01$$

stat. chiral ext., disc.,pert.,κ_s

Grinstein ratio is a useful quantity for the determination of CKM elements $|V_{ts}|/|V_{td}|$ from TeVatron and CLEO-c experimental data. Lattice QCD can provide the value of R_1 more precisely than the chiral perturbation theory.

CLEO-c plans to measure f_{D_s}, f_{D_d} with 2% accuracy.

$$\Rightarrow (f_{B_s}/f_{B_d}) = (\frac{f_{B_s}}{f_{B_d}} / \frac{f_{D_s}}{f_{D_d}}) \times (f_{D_s}/f_{D_d})^{CLEO-c}$$

Summary

- The determination of $|V_{ub}|$ suffers from the limited kinematical region in lattice calculation as well as large statistical and systematic errors.
- Using perturbative QCD and dispersive bounds we can improve the accuracy $|V_{ub}$.
- The determination of $|V_{td}|/|V_{ts}|$ suffers from large errors from the chiral extrapolation.
- Using the Grinstein ratio we can extract $|V_{td}|/|V_{ts}|$ more precisely owing to the approximate heavy quark symmetry and the SU(3) flavor symmetry.
- In both cases, more precise experimental data will help.
  $B \to \pi l \nu$ spectrum from B factories. D meson decay constants from charm factories.

Backup Slides

The dispersive bound

Consider the vacuum polarization function with ub current $V^{\mu\nu} \equiv \bar{u}\gamma^{\mu}b$ given by,

$$\Pi^{\mu\nu} \quad = i \int d^4x e^{iqx} \langle 0|T[V^{\mu}(x)V^{\nu\dagger}(0)]|0\rangle \quad = -(g^{\mu\nu}q^2 - q^{\mu}q^{\nu})\Pi_T(q^2) + q^{\mu\nu}\Pi_L(q^2).$$

transverse part longditudinal part

From the optical theorem, the imaginary part can be expressed by the sum of all the intermidiate hadronic states.

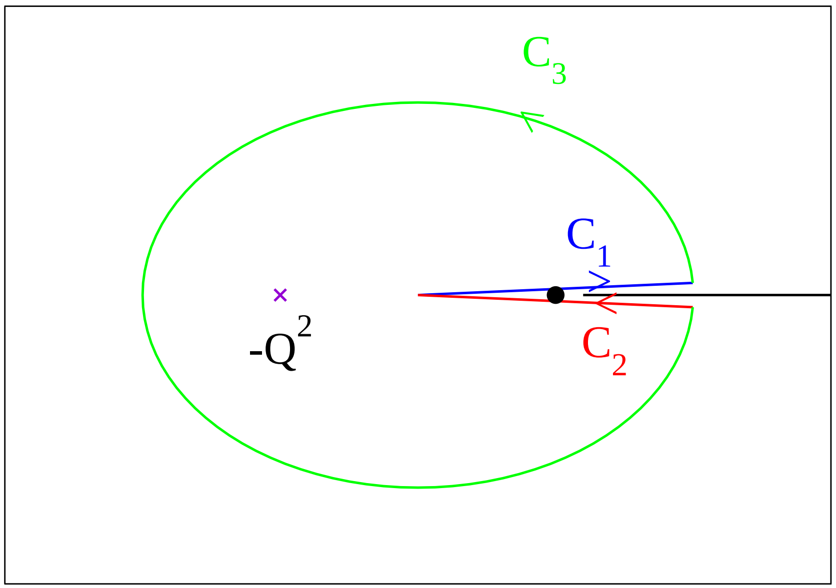
$$\mathrm{Im}\Pi^{\mu\nu}(q) \quad = \quad \frac{1}{2}\sum_{\Gamma}(2\pi)^4\delta^4(q - p_{\Gamma})\langle 0|V^{\mu}|\Gamma\rangle\langle\Gamma|V^{\nu\dagger}|0\rangle, \tag{5}$$

where $|\Gamma\rangle$ denotes B^*, $B\pi$ and all possible hadron states created by V^{μ}.
L.H.S. is calculable by PQCD if q is far from the resoncance, R.H.S contains $\langle 0|V^{\mu}|B\pi\rangle$.

Cauchy's theorem for complex functions $\int_C \frac{dz}{2\pi i}\frac{f(z)}{(z-a)^2} = f'(a)$

$$\chi(Q^2) \equiv \frac{d}{dt}(t\Pi(t))|_{t=-Q^2} = \int_{C_1}\frac{dt}{2\pi i}\frac{t\Pi(t)}{(t+Q^2)^2} + \int_{C_2}\frac{dt}{2\pi i}\frac{t\Pi(t)}{(t+Q^2)^2} + \int_{C_3}\frac{dt}{2\pi i}\frac{t\Pi(t)}{(t+Q^2)^2}$$

$$= \int_0^\infty \frac{dt}{\pi}\frac{t\,\mathrm{Im}\Pi(t)}{(t+Q^2)^2}$$ $\cdots$ once subtracted dispersion relation



$\chi(Q^2)$ is calculable by PQCD if $-Q^2$ is far from the resonance.
This equation holds for Π_L and Π_T.

Since all the states give positive contributions, if we just take $|B^*\rangle$, $|B\pi\rangle$ states and drop all the excited states, we obtain an EXACT INEQUALITY

$$\mathrm{Im}\Pi_L(t) \geq \frac{3}{2}\frac{t_+t_-}{16\pi}\sqrt{(t-t_+)(t-t_-)}\frac{|f^0(t)|^2}{t^3}\theta(t-t_+)$$

$$\mathrm{Im}\Pi_L(t) \geq \pi(\frac{m_{B^*}}{f_{B^*}})^2\delta(t-m_{B^*}) + \frac{3}{2}\frac{t_+t_-}{48\pi}[(t-t_+)(t-t_-)]^{3/2}\frac{|f^+(t)|^2}{t^3}\theta(t-t_+),$$

where $t \equiv q^2$, $t_\pm \equiv (m_B \pm m_\pi)^2$. Using the dispersion relatoins we obtain,

$$\chi_L(Q^2) \geq \frac{1}{\pi}\int_{t_+}^{\infty} k_L^0(t,Q^2)|f^0(t)|^2 \quad (6)$$

$$\chi_T(Q^2) \geq (\frac{m_{B^*}}{f_{B^*}})^2 + \frac{1}{\pi}\int_{t_+}^{\infty} k_L^+(t,Q^2)|f^+(t)|^2 \quad (7)$$

Both equations can be written in the form as $J(Q^2) > \frac{1}{\pi}\int_{t_+}^{\infty} k^+(t,Q^2)|f(t)|^2$

Making the change of variables as $t \longrightarrow z$ where $\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}$,

$$J(Q^2) \geq \int_{|z|=1} \frac{dz}{2\pi i z} |\phi(z, Q^2) f(z)|^2 = \langle \phi f | \phi f \rangle \tag{8}$$

where we defined the innerprodcut as $\langle g|h \rangle = \int_{|z=1|} \frac{dz}{2\pi i z} \overline{g(z)} h(z)$

The function $g_t(z) \equiv \frac{1}{1 - \overline{z(t)} z}$ can be used to extract the form factor at $z(t)$ as

$$\langle g_t | \phi f \rangle \quad = \quad \phi(z(t), Q^2) f(t) \tag{9}$$

$$\phi(z, Q^2) \quad = \quad \sqrt{\frac{2t_+ t_-}{4\pi}} \frac{1}{t_+ - t_-} \frac{1+z}{(1-z)^{5/2}} (\beta(0) + \frac{1+z}{1-z})^{-2} (\beta(-Q^2) + \frac{1+z}{1-z})^{-2}, \tag{10}$$

where $\beta = \sqrt{(t_+ - t)/(t_+ - t_-)}$.

Perturbative QCD results

$$\chi_L(Q^2) = \frac{1}{\pi m_b^2}\int_0^1 dx \frac{(m_b^2/x) Im\Pi_L^{pert}(x)}{(1+(Q^2/m_b^2)x)^2} + \frac{m_b\langle\bar{u}u\rangle_{1GeV}}{(Q^2+m_b^2)^2} + \frac{1}{(Q^2+m_b^2)^2}\langle\frac{\alpha_s}{12\pi}G^2\rangle,$$

$$\chi_T(Q^2) = \frac{1}{\pi m_b^2}\int_0^1 dx \frac{(m_b^2/x) Im\Pi_T^{pert}(x)}{(1+(Q^2/m_b^2)x)^3} + \frac{m_b\langle\bar{u}u\rangle_{1GeV}}{(Q^2+m_b^2)^3} + \frac{1}{(Q^2+m_b^2)^3}\langle\frac{\alpha_s}{12\pi}G^2\rangle,$$

where

$$Im\Pi_L^{pert}(x) = \frac{3}{8\pi}x(1-x)^2[1+\mathcal{O}(\alpha_f)],$$

$$Im\Pi_T^{pert}(x) = \frac{1}{8\pi}(1-x)^2[(2+x)+\mathcal{O}(\alpha_f)],$$

1-loop results by L. J. Reinders *et al.*, Phys. Lett. B 334 (1994) 175.

$$\chi_L(Q^2=0\text{GeV}^2) = 1.5\times 10^{-2}, \qquad \chi_T(Q^2=0\text{GeV}^2) = 5.6\times 10^{-4}.\text{at 1-loop},$$

where $m_b = 4.3$ GeV, $\langle\bar{u}u\rangle_{1GeV} = (-0.24GeV)^3$, $\langle\frac{\alpha_s}{\pi}G^2\rangle = 0.02(1)GeV^4$, are used.

Scheme dependence is tiny (perturbative correction is undercontrol).

Power corrections from condensates are tiny.