# Particle correlation studies in RHIC BES; probes for the critical end point in the QCD phase diagram

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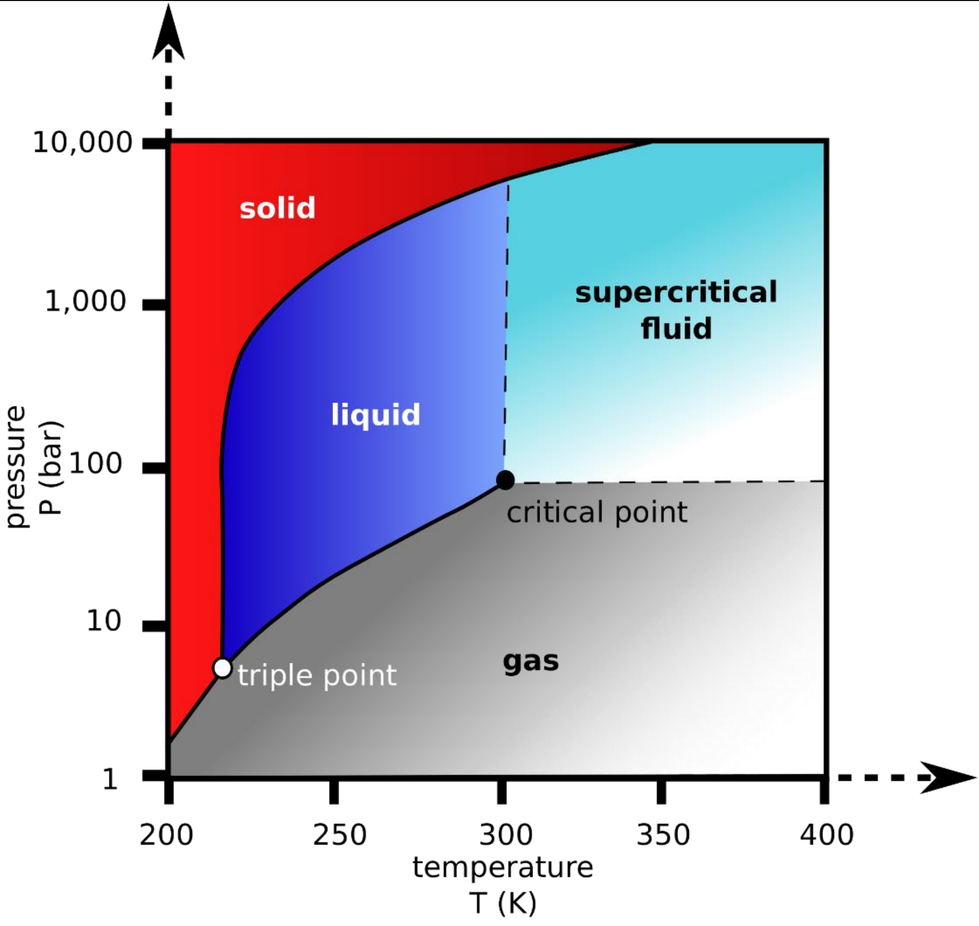
**Outline**

- ***Introduction***
  - *Phase Diagram & HIC*
- ***Search strategy for the CEP***
  - *Theoretical guidance*
  - *Guiding principles for search*
- ***The probe***
  - *Femtoscopic "susceptibility"*
- ***Analysis***
  - *Finite-Size-Scaling*
  - *Dynamic Finite-Size-Scaling*
- ***Summary***
  - ***Epilogue***

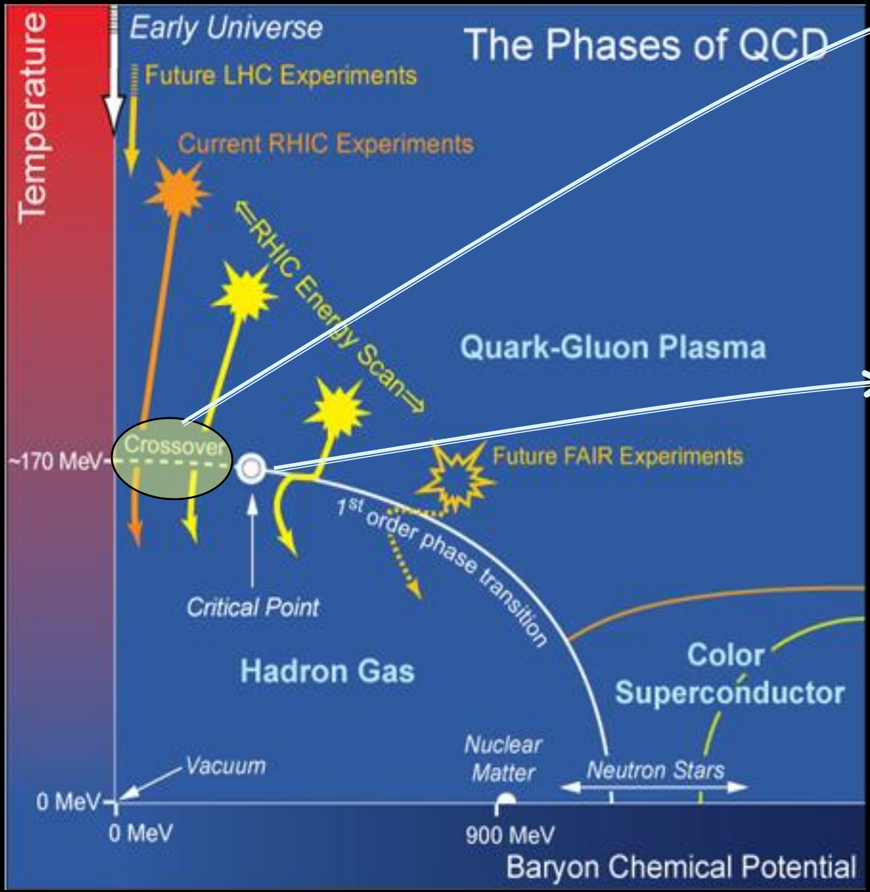
# Characterizing the phases of matter

**The location of the critical End point and the phase coexistence regions are fundamental to the phase diagram of any substance !**

*The properties of each phase is also of fundamental interest*



# The QCD Phase Diagram




*Known knowns*

*Spectacular achievement:*

- *Validation of the crossover transition leading to the QGP*
- *Initial estimates for the transport properties of the QGP*

*Known unknowns*

- **Location of the critical point (CEP)?**
  - ✓ **Order of the phase transition?**
  - ✓ **Value of the critical exponents?**
- **Location of phase coexistence regions?**
- **Detailed properties of each phase?**

**All are fundamental to charting the phase diagram**

*(New) measurements, analysis techniques and theory efforts which probe a broad range of the **(T, $\mu_B$)-plane,** are essential to fully unravel the unknowns!*

- RHIC

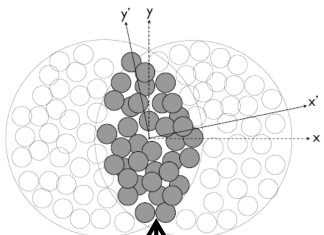

- LHC

- First collisions 2000
- p+p, d+Au, Cu+Cu, Cu+Au, Au+Au, U+U
- $\sqrt{s_{NN}}$ ~ 7 – 200 GeV

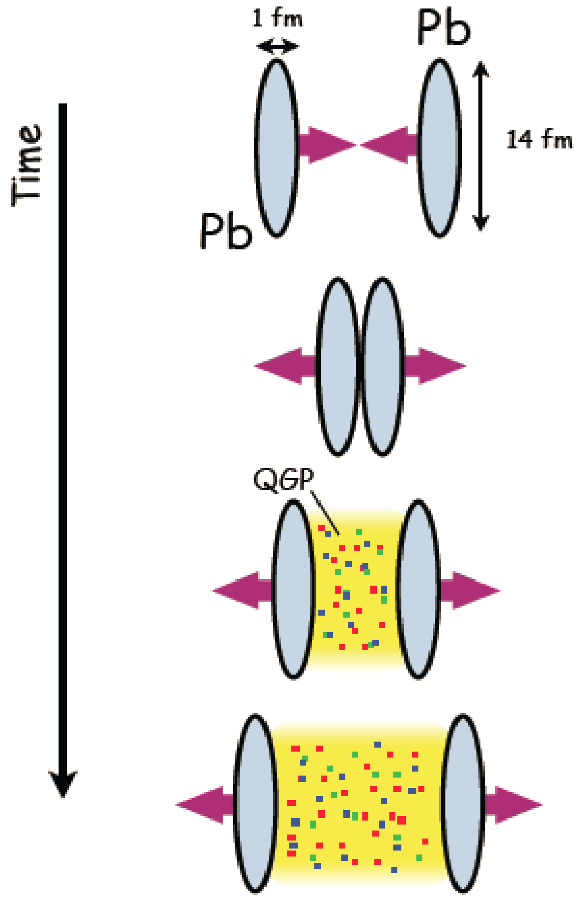
- First collisions 2010
- p+p, Pb+Pb, p+Pb
- $\sqrt{s_{NN}}$ =2.76 TeV
- (5.5 TeV in 2015-16)

***The RHIC and LHC are two major experimental facilities currently used to study the QCD phase diagram***

# Heavy Ion Collisions




# participants
$N_{part}$




Heavy Ion Collision:
Ideal to get conditions at high T and ρ

Formation time $\tau_0 = 1$ fm/c
[1 fm/c = $3.3 \times 10^{-24}$ s]

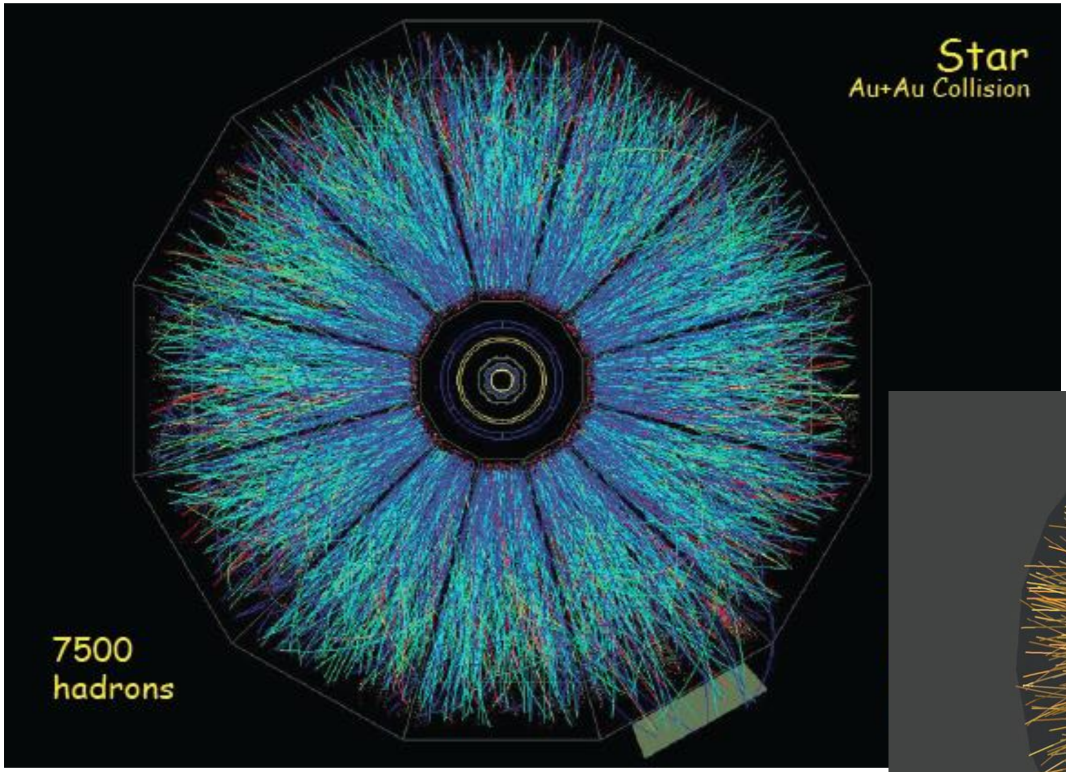
Temperature of $O(10^{12})$
Lifetime: 10 fm/c

QGP in equilibrium (!?)
Cool down: hadronization

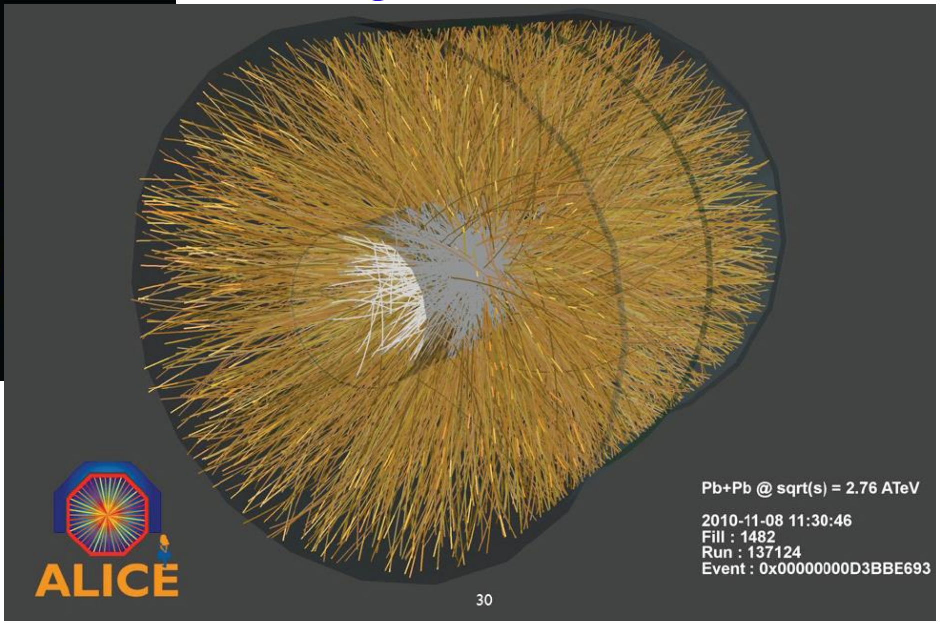
***Heavy ion collisions are used to produce the hot and dense matter used to probe the QCD phase diagram***

# A Typical Event

- **RHIC**

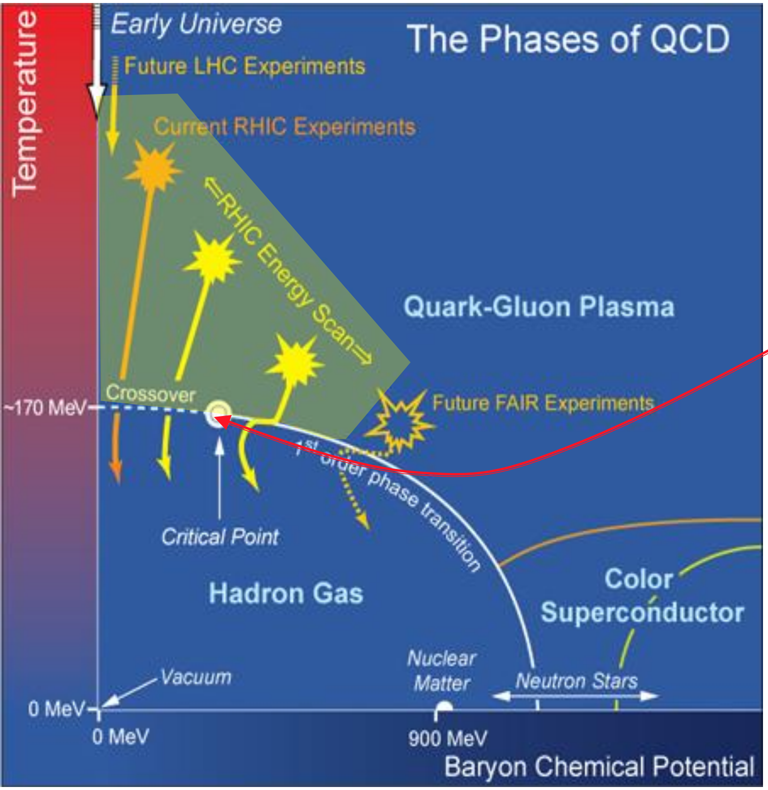



- **LHC**




***The particles produced in collision events are used to study the produced medium***

# The QCD Phase Diagram




## Essential Question

**What new insights do we have on:**

***The CEP "landmark"?***

- ✓ *Location ($T^{cep}, \mu_B^{cep}$) values?*
- ✓ *Static critical exponents - $\nu, \gamma$?*
  - *Static universality class?*
  - *Order of the transition*
- ✓ *Dynamic critical exponent – z?*
  - *Dynamic universality class?*

**All are required to fully characterize the CEP & drives the ongoing search**

# Theoretical Guidance

**Theory consensus on the static universality class for the CEP**

*3D-Ising* Z(2)

- ✓ $\nu \sim 0.63$
- ✓ $\gamma \sim 1.2$

**M. A. Stephanov**
**Int. J. Mod. Phys. A 20, 4387 (2005)**

***Dynamic Universality class for the CEP less clear***

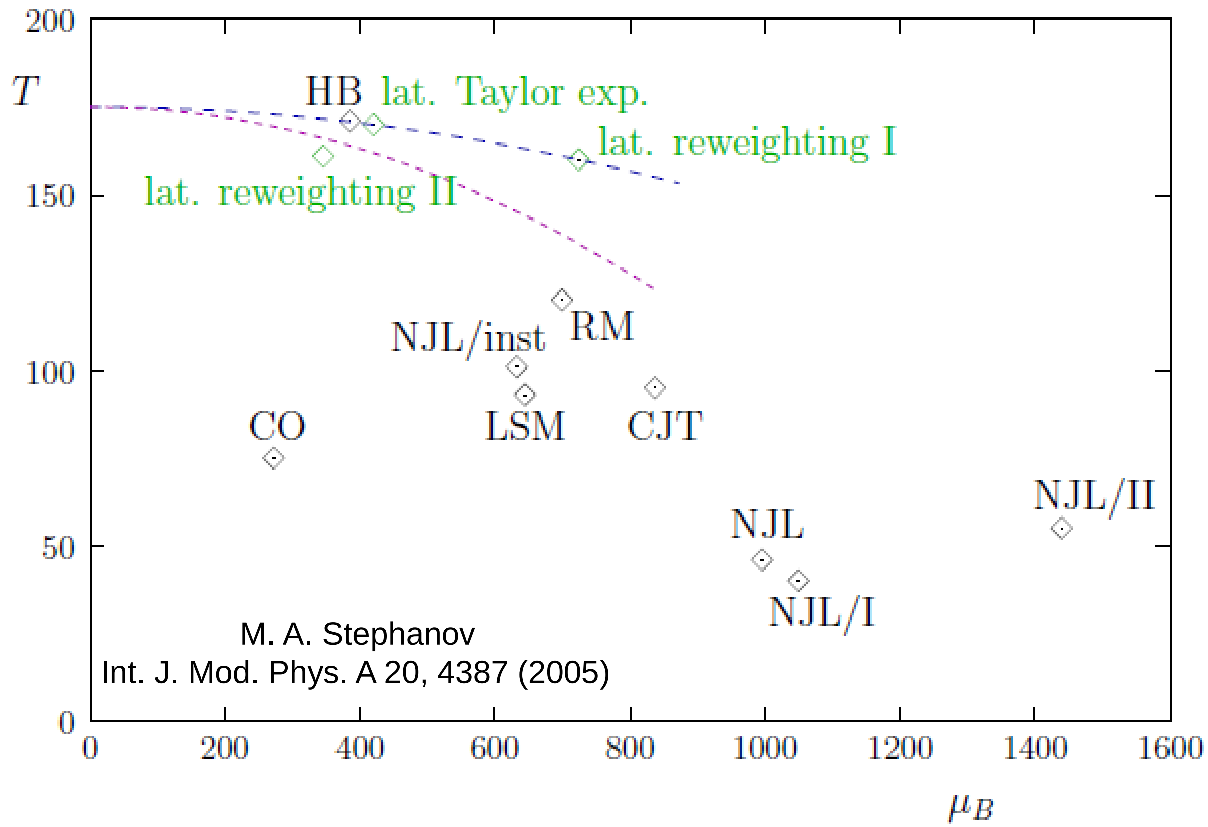
- **One slow mode**
  - ✓ ***z ~ 3 - Model H***

**Son & Stephanov**
**Phys.Rev. D70 (2004) 056001**
**Moore & Saremi ,**
**JHEP 0809, 015 (2008)**

- **Three slow modes**
  - ✓ $z_T \sim 3$
  - ✓ $z_v \sim 2$
  - ✓ $z_s \sim -0.8$

*Y. Minami* - **Phys.Rev. D83 (2011) 094019**

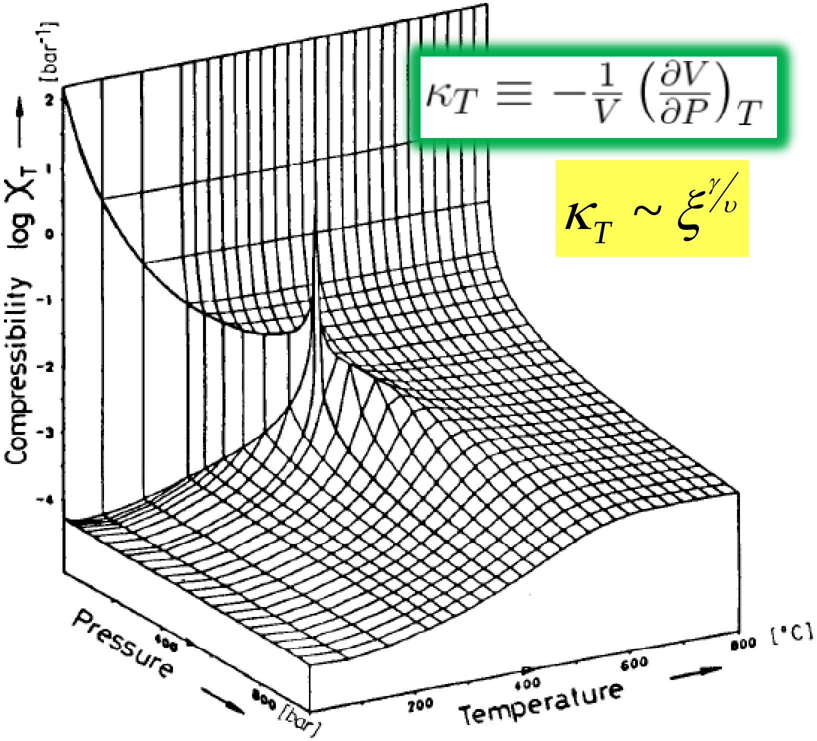
**The predicted location ($T^{cep}, \mu_B^{cep}$) of the CEP is even less clear*!***




***Experimental verification and characterization of the CEP is a crucial ingredient***

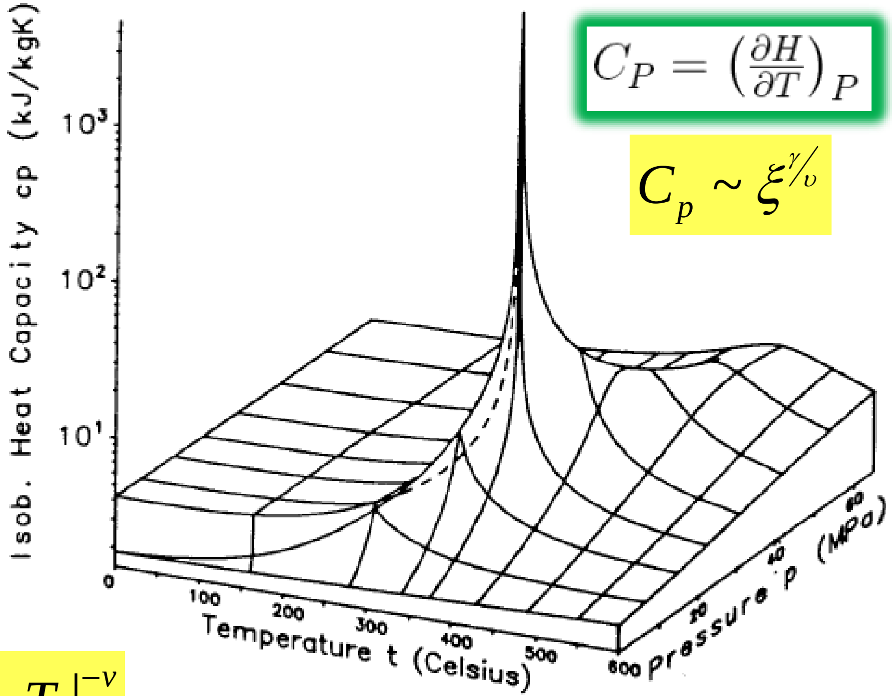
# *Anatomy of search strategy*

$H_2O$




$$\kappa_T \equiv -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

$$\kappa_T \sim \xi^{\gamma/\upsilon}$$




$$C_P = \left(\frac{\partial H}{\partial T}\right)_P$$

$$C_p \sim \xi^{\gamma/\upsilon}$$

$$\xi \sim |T - T_c|^{-\nu}$$

- $\langle(\delta n)\rangle \sim \xi^2$ → ***search for "critical" fluctuations in HIC***

***Stephanov, Rajagopal, Shuryak, PRL.81, 4816 (98)***

**The critical point is characterized by several (power law) divergences**

**Central idea → use beam energy scans to vary $\mu_B$ & *T to search for the influence of such divergences!***

***→ We use femtoscopic measurements to perform our search***

# *Interferometry as a susceptibility probe*

**S. Afanasiev et al. (PHENIX)**
**PRL 100 (2008) 232301**

*Hanbury Brown & Twist (HBT) radii obtained from two-particle correlation functions*

$$C(\mathbf{q}) = \frac{dN_2 / d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1/d\mathbf{p}_1)(dN_1/d\mathbf{p}_2)}$$

**3D Koonin Pratt Eqn.**

$$R(\vec{q}) = C(\vec{q}) - 1 = 4\pi \int dr r^2 K_0(\vec{q},\vec{r}) S(\vec{r}) \quad (1)$$

**Correlation function**

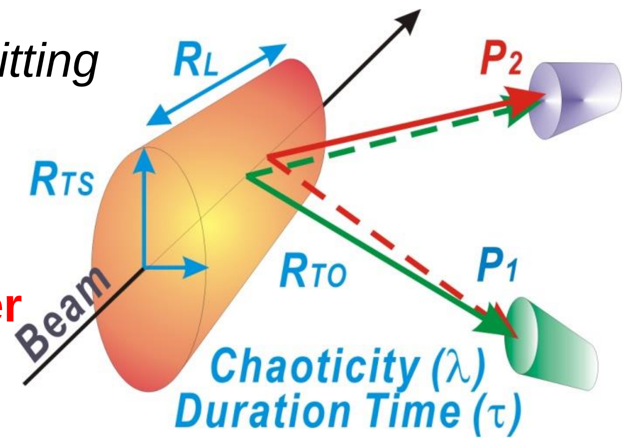
**Encodes FSI**

**Source function**
**(Distribution of pair separations)**

**Inversion of this integral equation ➔ Source Function**
$(R_L, R_{To}, R_{Ts})$

*The expansion of the emitting source $(R_L, R_{To}, R_{Ts})$ produced in HI collisions is driven by $c_s$*

**χ of the order parameter diverges at the CEP**



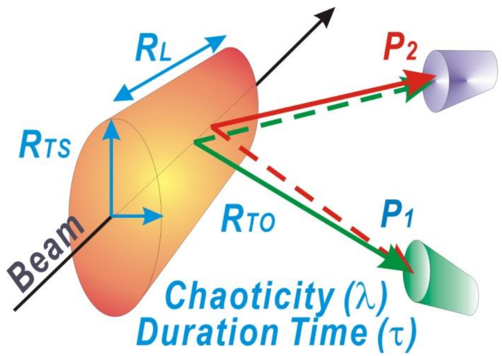

$$c_s^2 = \frac{1}{\rho\kappa}$$ ← ***Susceptibility (χ)***

***In the vicinity of a phase transition or the CEP, the divergence of κ leads to anomalies in the expansion dynamics***

***Strategy***

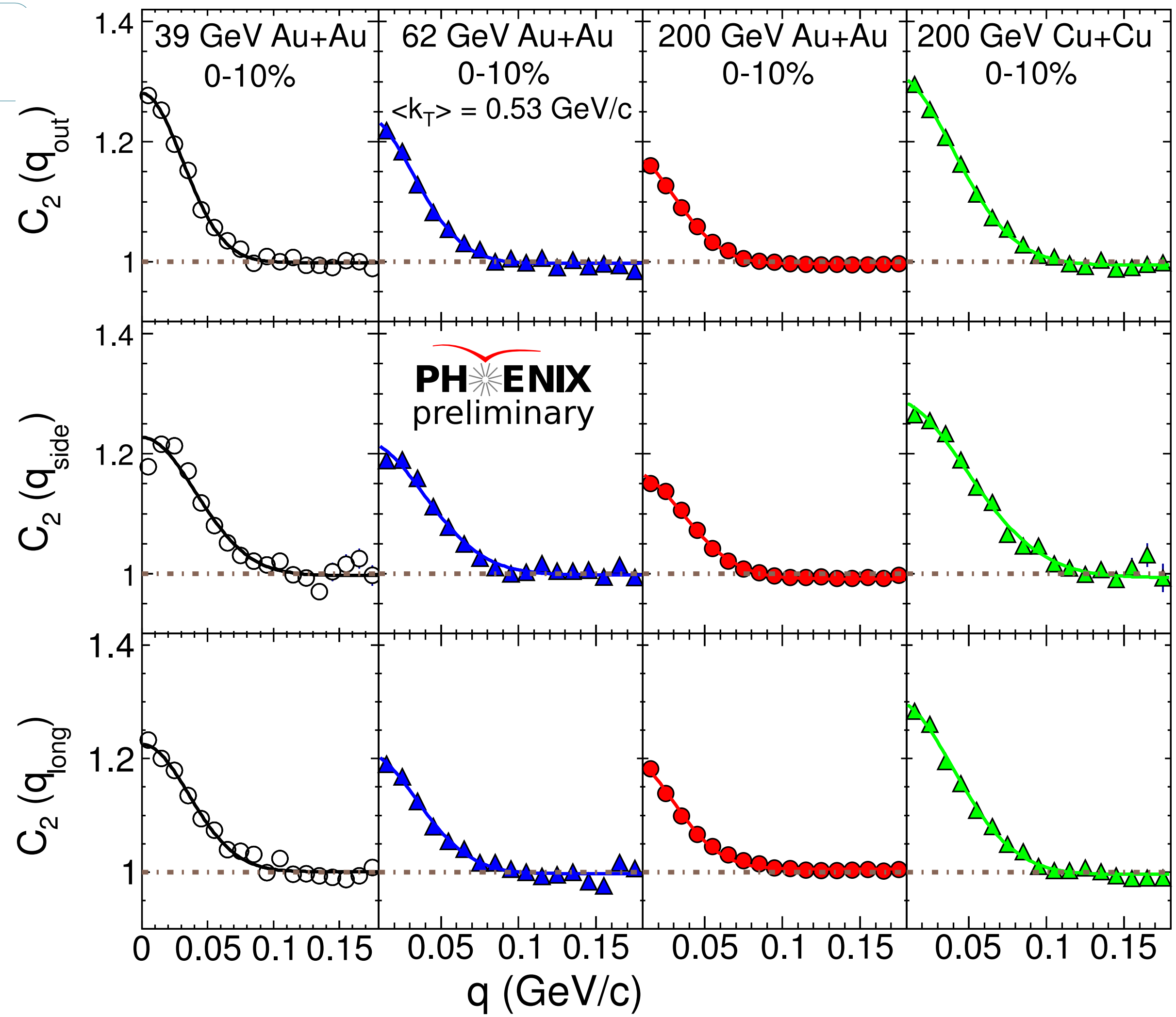
***Search for non-monotonic patterns for HBT radii combinations that are sensitive to the divergence of κ***

# Interferometry signal




$$C(\mathbf{q}) = \frac{dN_2/d\mathbf{p}_1 d\mathbf{p}_2}{(dN_1/d\mathbf{p}_1)(dN_1/d\mathbf{p}_2)}$$

**Adare et. al. (PHENIX)**
arXiv:1410.2559




$$C_2(\mathbf{q}) = N[(\lambda(1 + G(\mathbf{q})))F_c + (1 - \lambda)],$$

$$G(\mathbf{q}) \cong \exp(-R^2_{\rm side}q^2_{\rm side} - R^2_{\rm out}q^2_{\rm out} - R^2_{\rm long}q^2_{\rm long})$$

***Strategy***

***Search for non-monotonic patterns for HBT radii combinations that are sensitive to the divergence of κ***

# Interferometry Probe

Hung, Shuryak, PRL. 75,4003 (95)
Chapman, Scotto, Heinz, PRL.74.4400 (95)

Makhlin, Sinyukov, ZPC.39.69 (88)

**The measured HBT radii encode space-time information for the reaction dynamics**

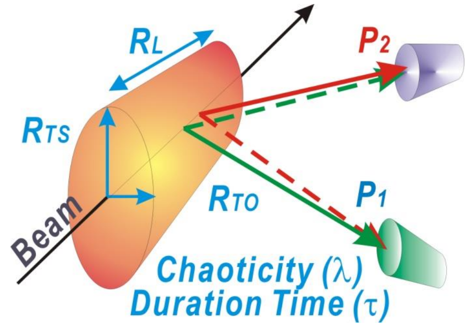
$$R_{side}^2 = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2}$$

emission duration

$$R_{out}^2 = \frac{R^2_{geo}}{1+\frac{m_T}{T}\beta_T^2} + \beta_T^2(\Delta\tau)^2$$

$$R_{long}^2 \approx \frac{T}{m_T}\tau^2$$

emission lifetime




$$c_s^2 = \frac{1}{\rho\kappa}$$

***The divergence of the susceptibility κ***

- ✓ *"softens" the sound speed $c_s$*
- ✓ *extends the emission duration*

**($R^2_{out}$ - $R^2_{side}$) sensitive to the κ**

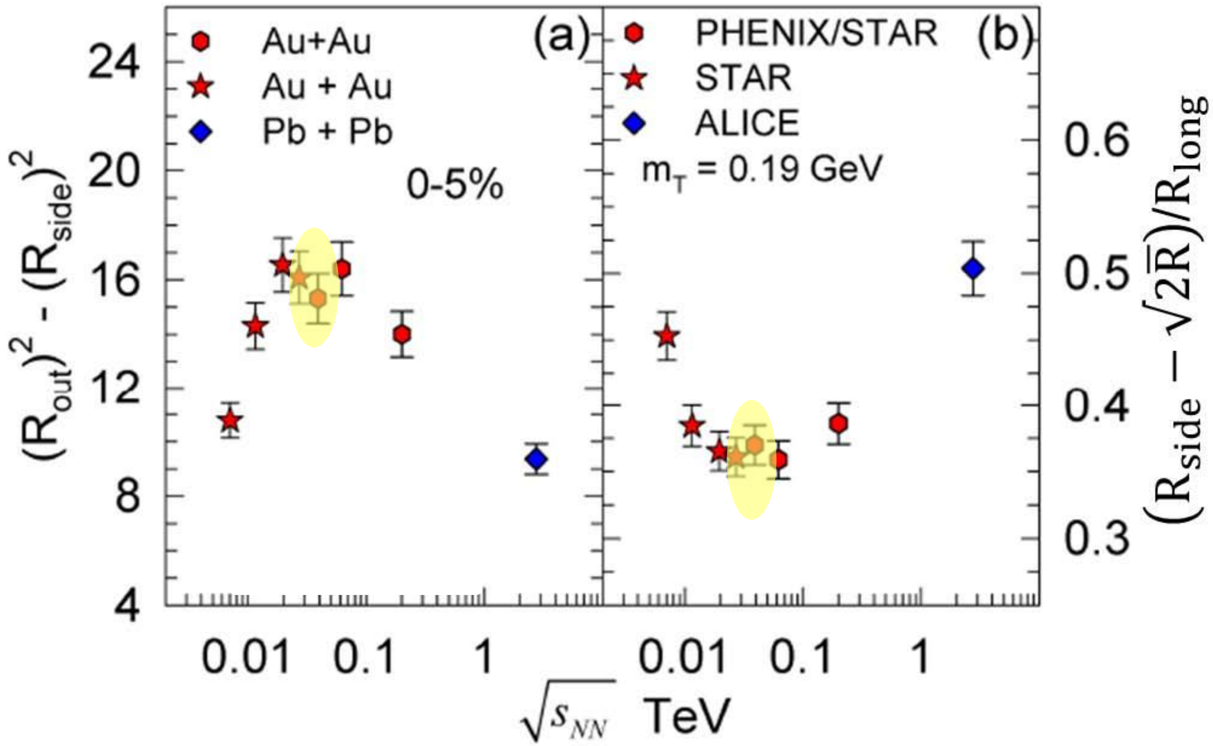
**($R_{side}$ - $R_{init}$)/ $R_{long}$ sensitive to $c_s$**

*Specific non-monotonic patterns expected as a function of $\sqrt{s_{NN}}$*

- **A maximum for ($R^2_{out}$ - $R^2_{side}$)**
- ***A minimum for* ($R_{side}$ - $R_{initial}$)/$R_{long}$**

# $\sqrt{s_{NN}}$ dependence of interferometry signal

Lacey QM2014
Adare et. al. (PHENIX)
arXiv:1410.2559




$R_{long} \propto \tau$

$\left(R_{out}^2 - R_{side}^2\right) \propto \Delta\tau^2$

$\left(R_{side} - R_i\right) / R_{long} \propto u$

$R_{initial} = \sqrt{2}\bar{R}$

**The measurements validate the expected non-monotonic patterns!**

**→ Reaction trajectories spend a fair amount of time near a "soft point" in the EOS that coincides with the CEP!**

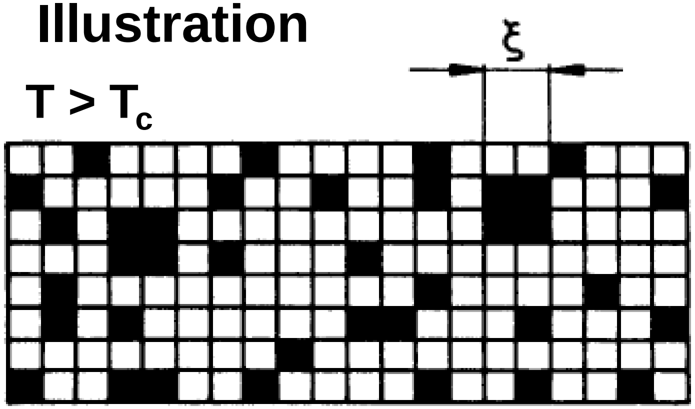
**** Note that $R_{long}$, $R_{out}$ and $R_{side}$ [all] increase with $\sqrt{s_{NN}}$ ****

**Finite-Size Scaling (FSS) is used for further validation of the CEP, as well as to characterize its static and dynamic properties**

# Basis of Finite-Size Effects

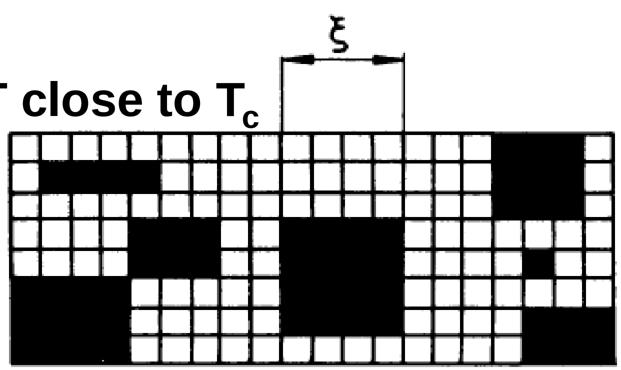
**Illustration**

**$T > T_c$**

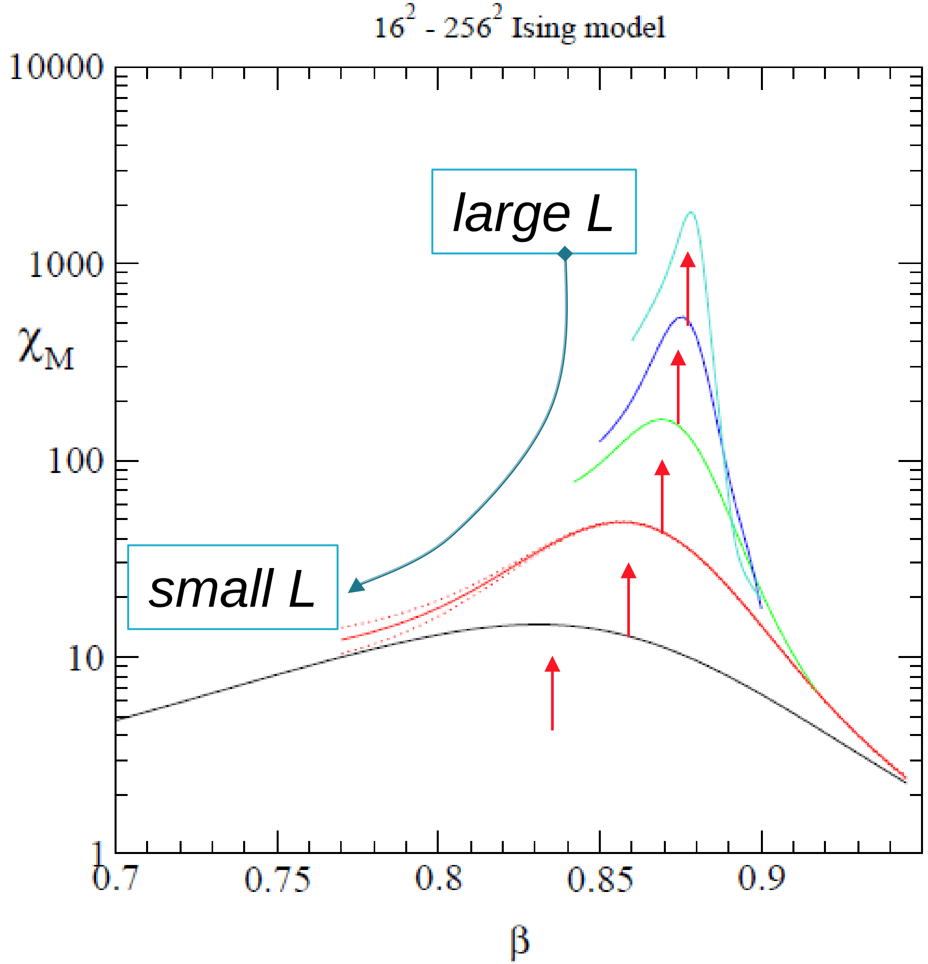



**L characterizes the system size**

**T close to $T_c$**




$$\xi \sim |T - T_c|^{-\nu} \leq L$$



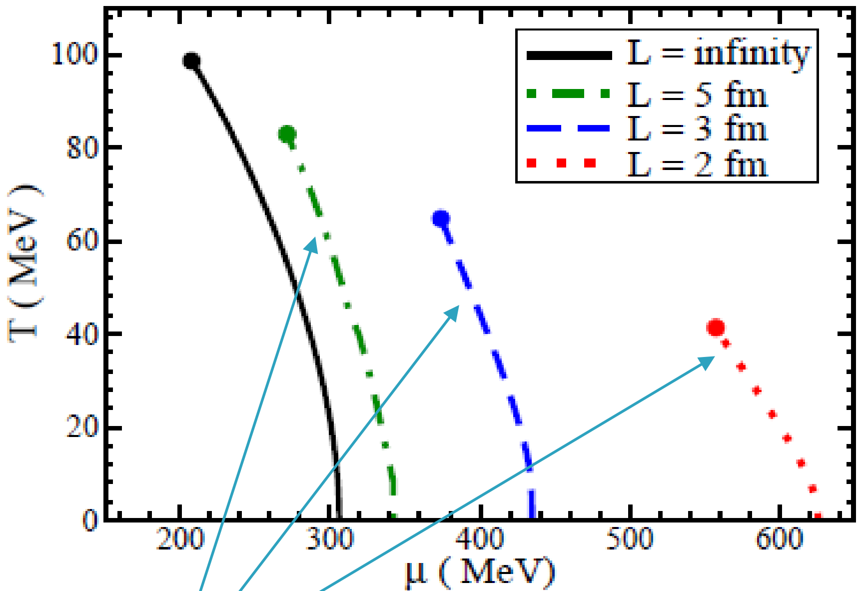

**note change in peak heights, positions & widths**

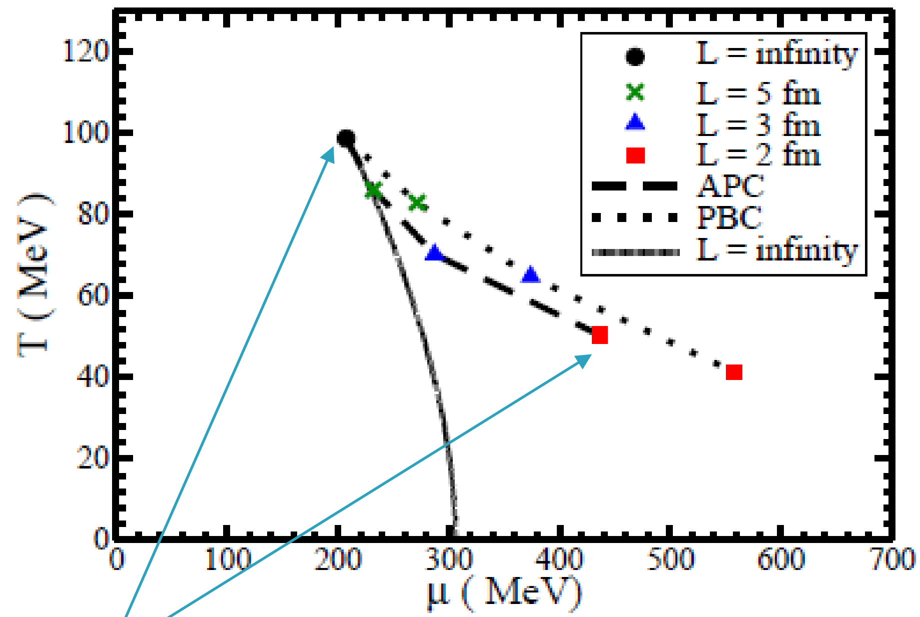
**→ A curse of Finite-Size Effects (FSE)**

**→ Only a pseudo-critical point is observed → shifted from the genuine CEP**

# *The curse of Finite-Size effects*

*E. Fraga et. al.*
**J. Phys.G 38:085101, 2011**




Displacement of pseudo-first-order transition lines and CEP due to finite-size

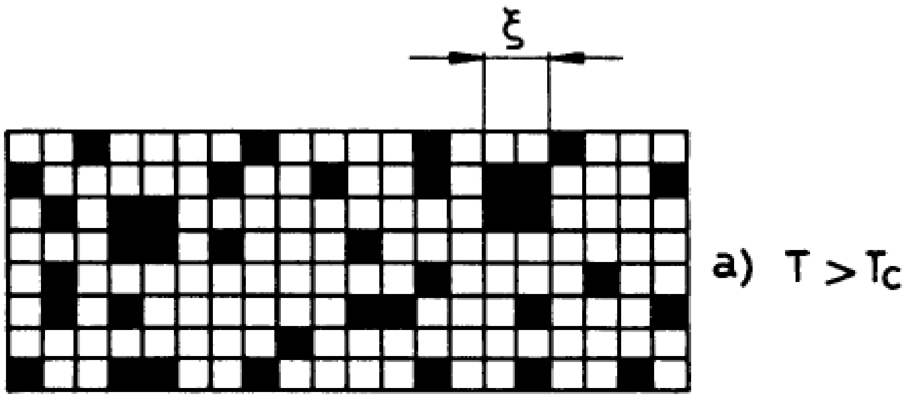
**Finite-size shifts both the pseudo-critical point and the transition line**

→ *A flawless measurement, sensitive to FSE,* ***can*** **not** *give the precise location of the CEP*

# The Blessings of Finite-Size

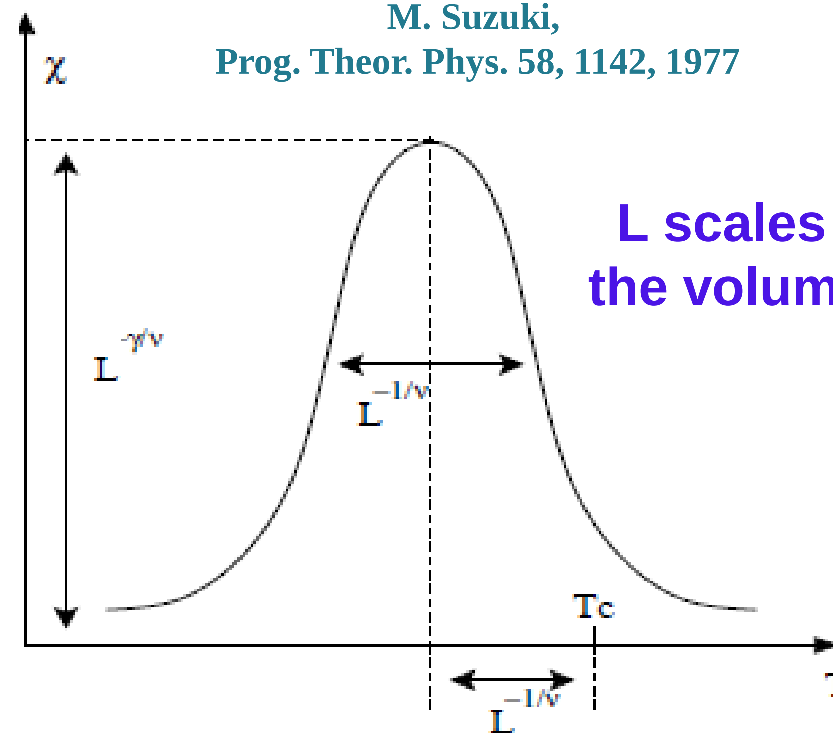
$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$
$$\tau_T(V) \sim T^{\rm cep}(V) - T^{\rm cep}(\infty) \sim L^{-\frac{1}{\nu}},$$

$$\chi(T,L) = L^{\gamma/\nu} P_\chi(tL^{1/\nu}) \qquad t = (T - T_c)/T_c$$

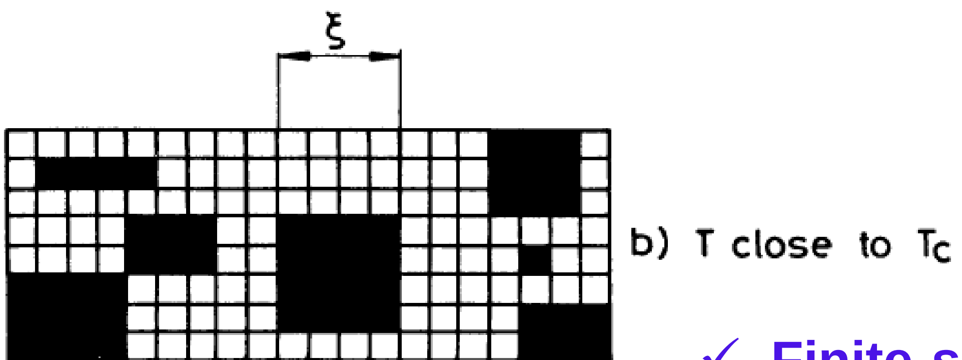



**Finite-size effects have a specific L dependence**

**M. Suzuki, Prog. Theor. Phys. 58, 1142, 1977**




**L scales the volume**



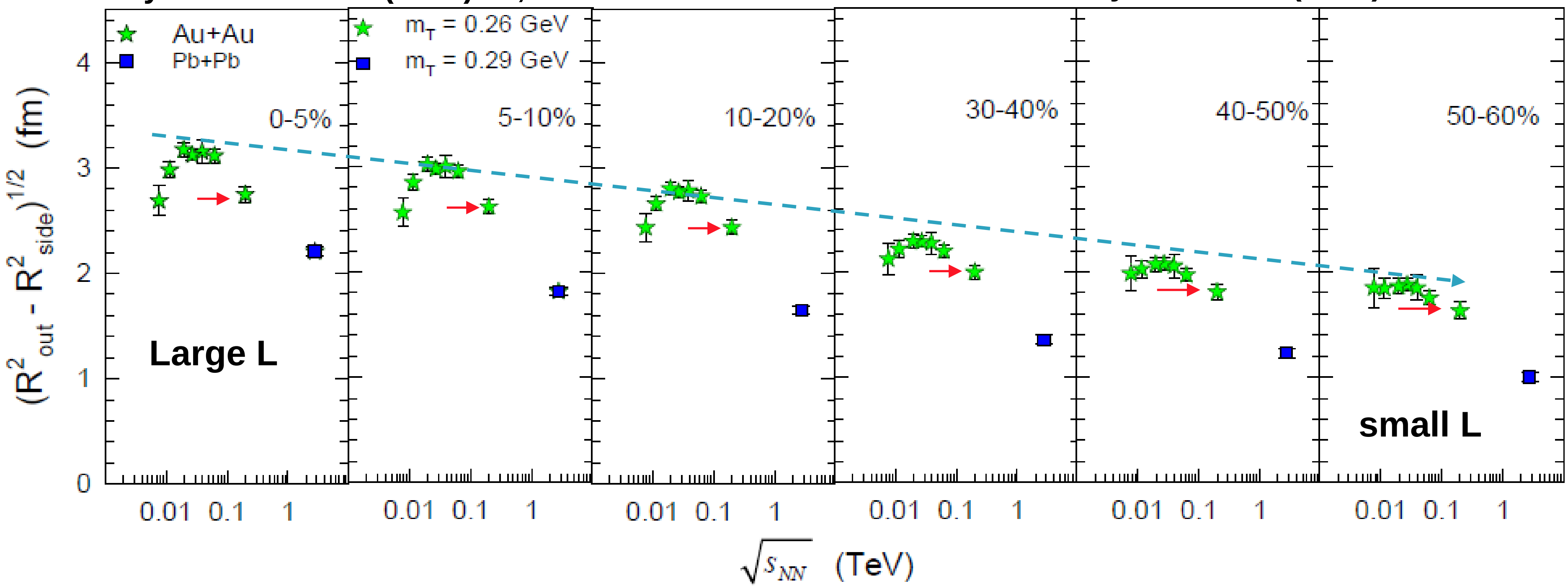

$$\xi \sim |T - T_c|^{-\nu} \le L$$

- ✓ **Finite-size effects have specific identifiable dependencies on size (L)**
- ✓ **The scaling of these dependencies give access to the CEP's location and it's critical exponents**

# Size dependence of HBT excitation functions

**Roy A. Lacey**
**Phys.Rev.Lett. 114 (2015) 14, 142301**

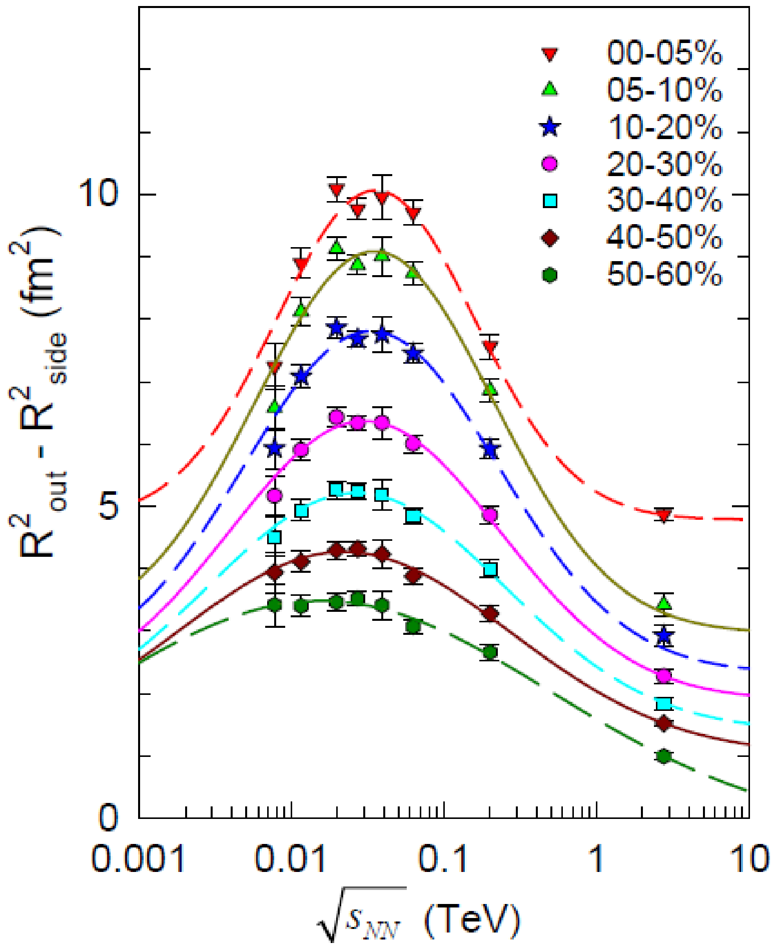
*Data from* L. Adamczyk et al. (STAR)
**Phys.Rev. C92 (2015) 1, 014904**

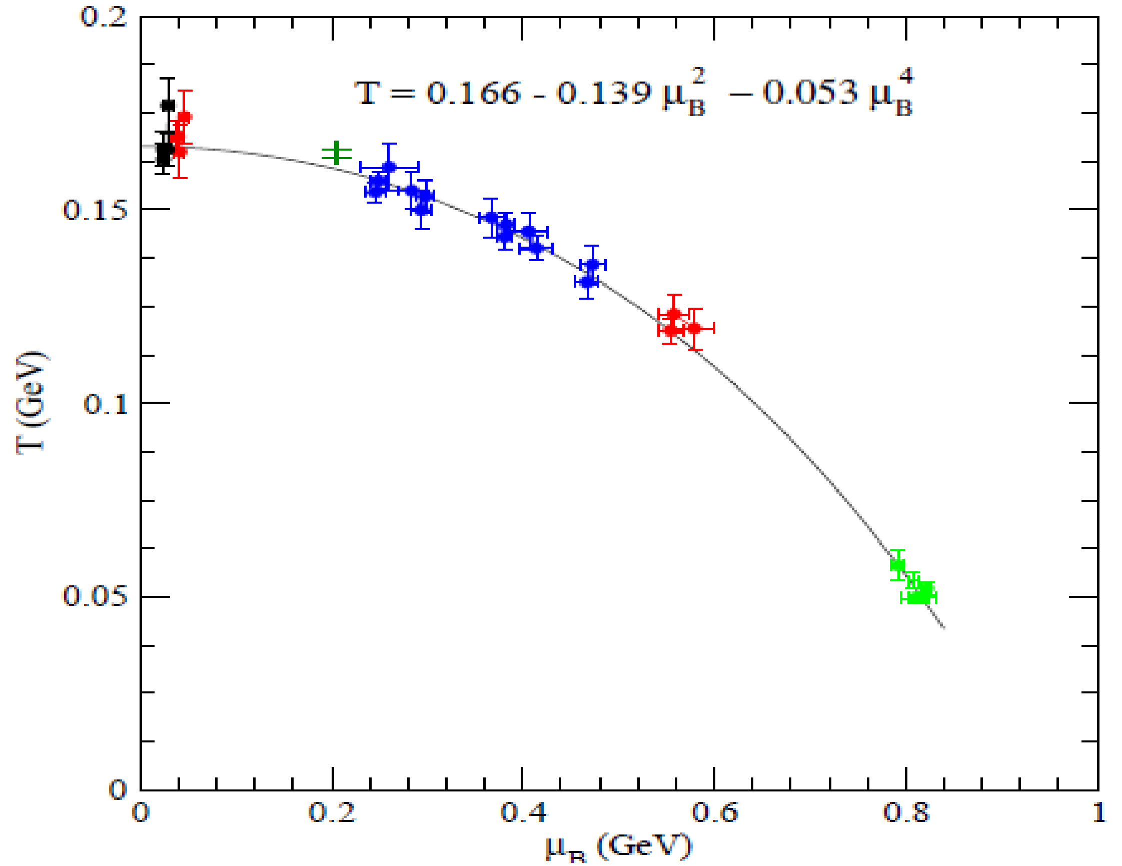



**The data validate the expected patterns for Finite-Size Effects**

- ✓ **Max values decrease with decreasing system size**
- ✓ **Peak positions shift with decreasing system size**
- ✓ **Widths increase with decreasing system size**

# *Size dependence of HBT excitation functions*



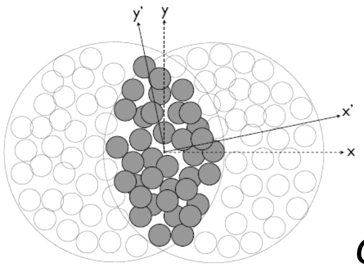

**characteristic patterns signal the effects of finite-size**

**II. Parameterize distance to the CEP by $\sqrt{s_{NN}}$**

$$\tau_s = \left(\sqrt{s} - \sqrt{s_{CEP}}\right)/\sqrt{s_{CEP}}$$

**III. Perform Finite-Size Scaling analysis with length scale $L = \bar{R}$**

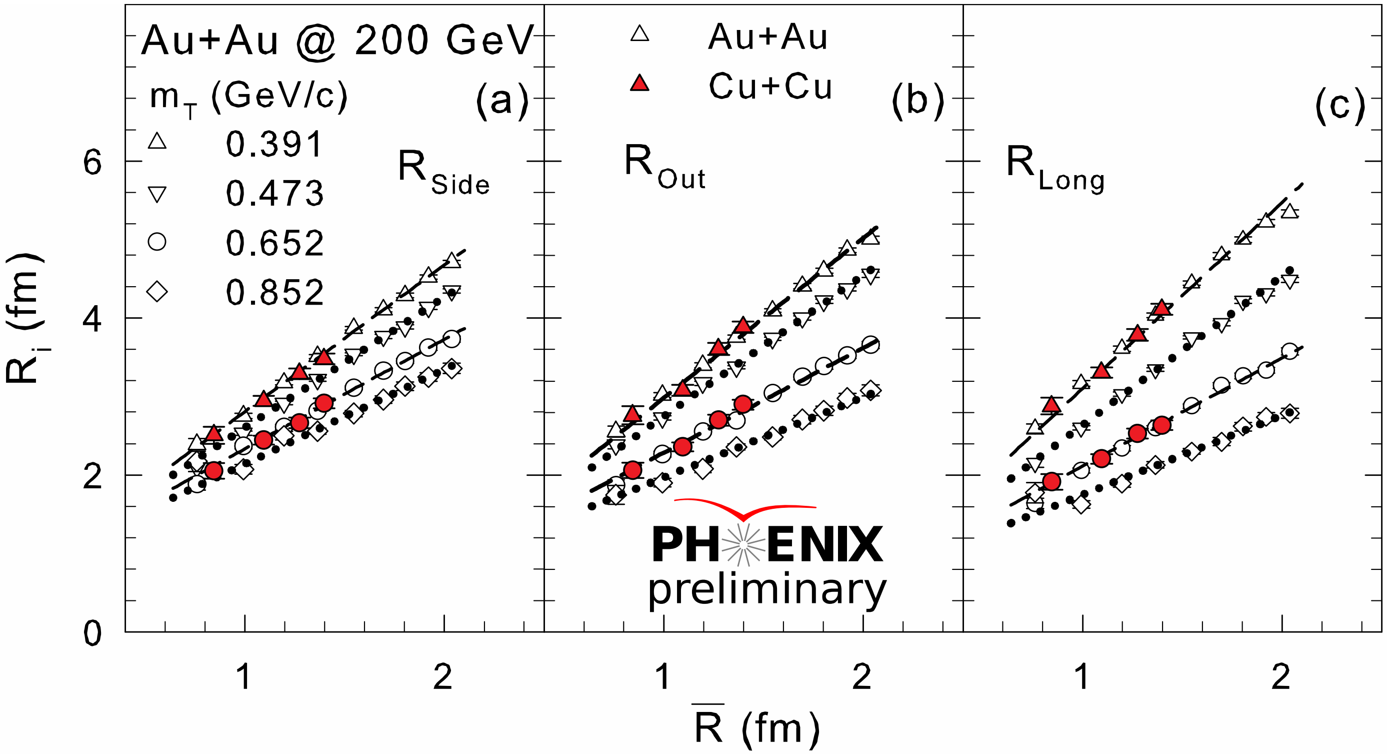
# Length Scale for Finite Size Scaling

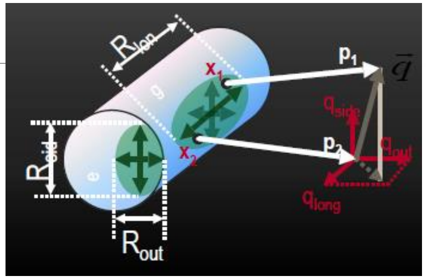


$$\frac{1}{\bar{R}} = \sqrt{\left(\frac{1}{\sigma_x^2} + \frac{1}{\sigma_y^2}\right)}$$

$\bar{R}$ is a characteristic length scale of the initial-state transverse size,

*$\sigma_x$ & $\sigma_y$ → RMS widths of density distribution*




$R_{out}, R_{side}, R_{long} \propto \bar{R}$

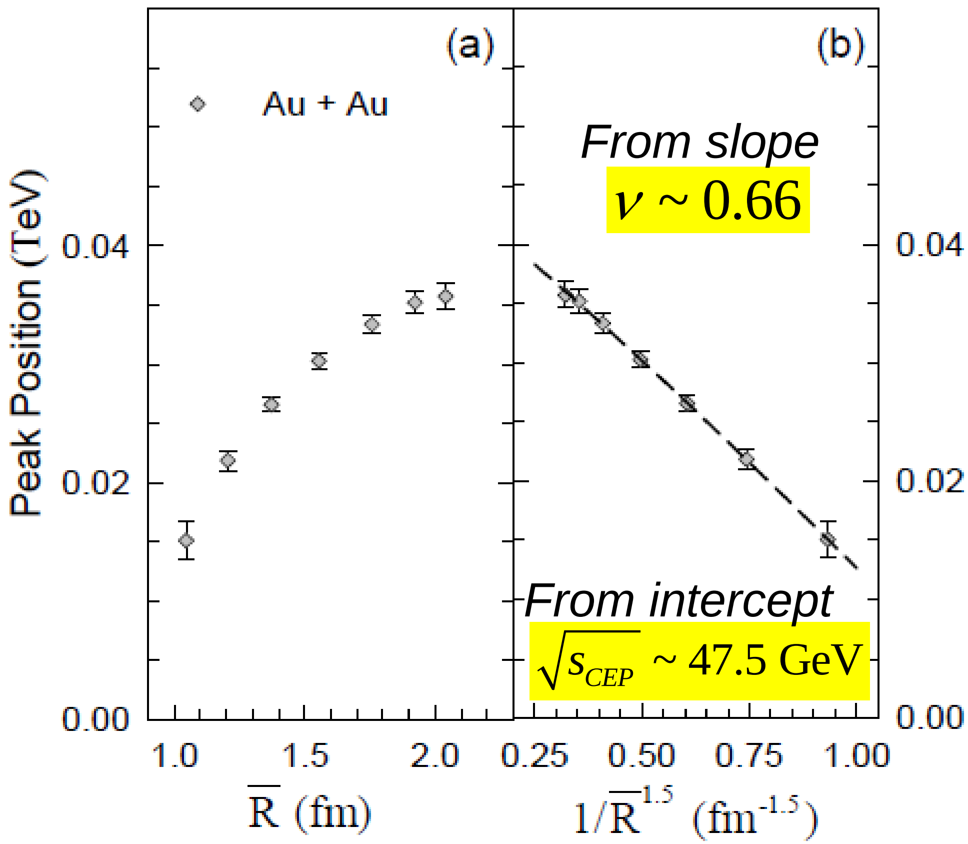
**$\bar{R}$ scales the volume**

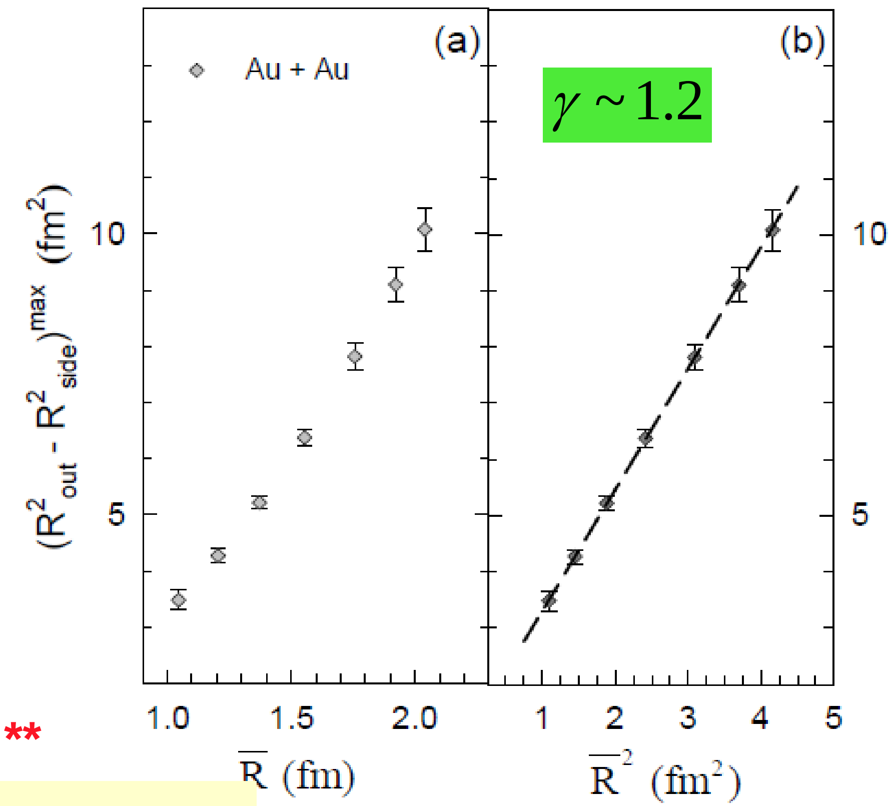
➤ ***$\bar{R}$ scales the full RHIC and LHC data sets***

# *Finite – Size Scaling*

$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-1/\nu}$$

$$\left(R^2_{out} - R^2_{side}\right)^{max} \propto \bar{R}^{\gamma/\nu}$$




**** Same $\nu$ value from analysis of the widths ****

- ***The critical exponents validate***
  - ✓ ***the 3D Ising model (static) universality class***
  - ✓ ***2nd order phase transition for CEP***

$$T^{cep} \sim 165 \text{ MeV}, \ \mu_B^{cep} \sim 95 \text{ MeV}$$

***($\sqrt{s_{CEP}}$ & chemical freeze-out systematics)***

# Closurer test for FSS

- ***$2^{nd}$ order phase transition***
- ***3D Ising Model (static) universality class for CEP***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

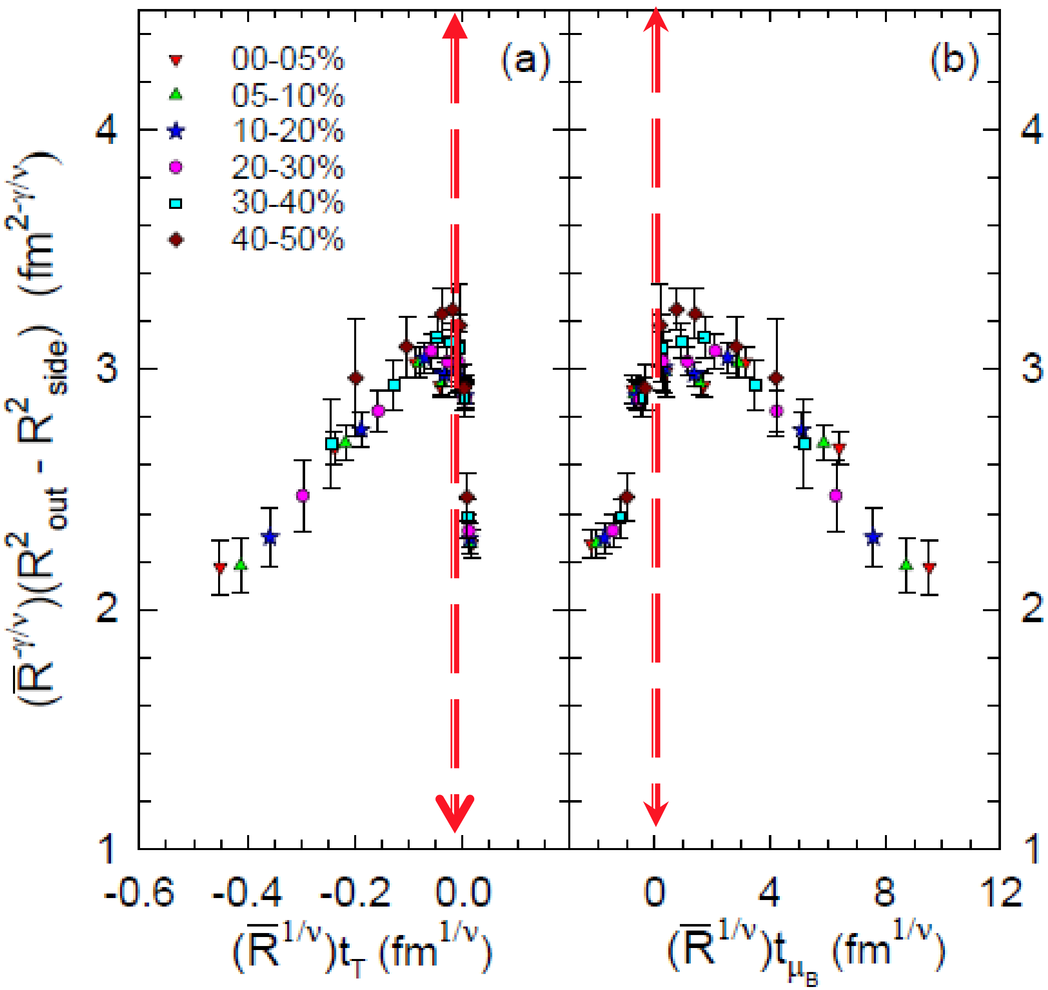
$$\chi(T,L) = L^{\gamma/\nu} P_\chi (tL^{1/\nu})$$

**M. Suzuki,**
**Prog. Theor. Phys. 58, 1142, 1977**

**Use $T^{cep}$, $\mu_B^{cep}$, ν and γ to obtain Scaling Function $P_\chi$**

$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_T,$$
$$\bar{R}^{-\gamma/\nu} \times (R^2_{\rm out} - R^2_{\rm side}) \text{ vs. } \bar{R}^{1/\nu} \times t_{\mu_B},$$
$$t_T = (T - T^{\rm cep})/T^{\rm cep}$$
$$t_{\mu_B} = (\mu_B - \mu_B^{\rm cep})/\mu_B^{\rm cep}$$

**T anf $\mu_B$ *are* from $\sqrt{s_{NN}}$**




*****A further validation of the location of the CEP and the (static) critical exponents*****

# What about Finite-Time Effects (FTE)?

$\chi_{op}$ diverges at the CEP
so relaxation of the order parameter could be anomalously slow

$\tau \sim \xi^{z}$ ← dynamic critical exponent

*z > 0 - Critical slowing down*

**Multiple slow modes?**

*$z_T \sim 3$, $z_v \sim 2$, $z_s \sim -0.8$*

*z < 0 - Critical speeding up*

*Y. Minami -* **Xiv:1201.6408**

**An important consequence**

$\xi \sim \tau^{1/z}$

*Significant signal attenuation for short-lived processes with $z_T \sim 3$ or $z_v \sim 2$*

***eg.*** $\langle(\delta n)\rangle \sim \xi^2$ *(without FTE)*
$\langle(\delta n)\rangle \ll \xi^2$ *(with FTE)*

***The value of the dynamic critical exponent/s is crucial for HIC***

**Dynamic Finite-Size Scaling (DFSS) is used to estimate the dynamic critical exponent z**

# *Dynamic Finite – Size Scaling*

****Experimental estimate of the dynamic critical exponent****

- ***2nd order phase transition***

$\nu \sim 0.66$ $\gamma \sim 1.2$

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

**DFSS ansatz**

*at time $\tau$ when T is near $T_{cep}$*

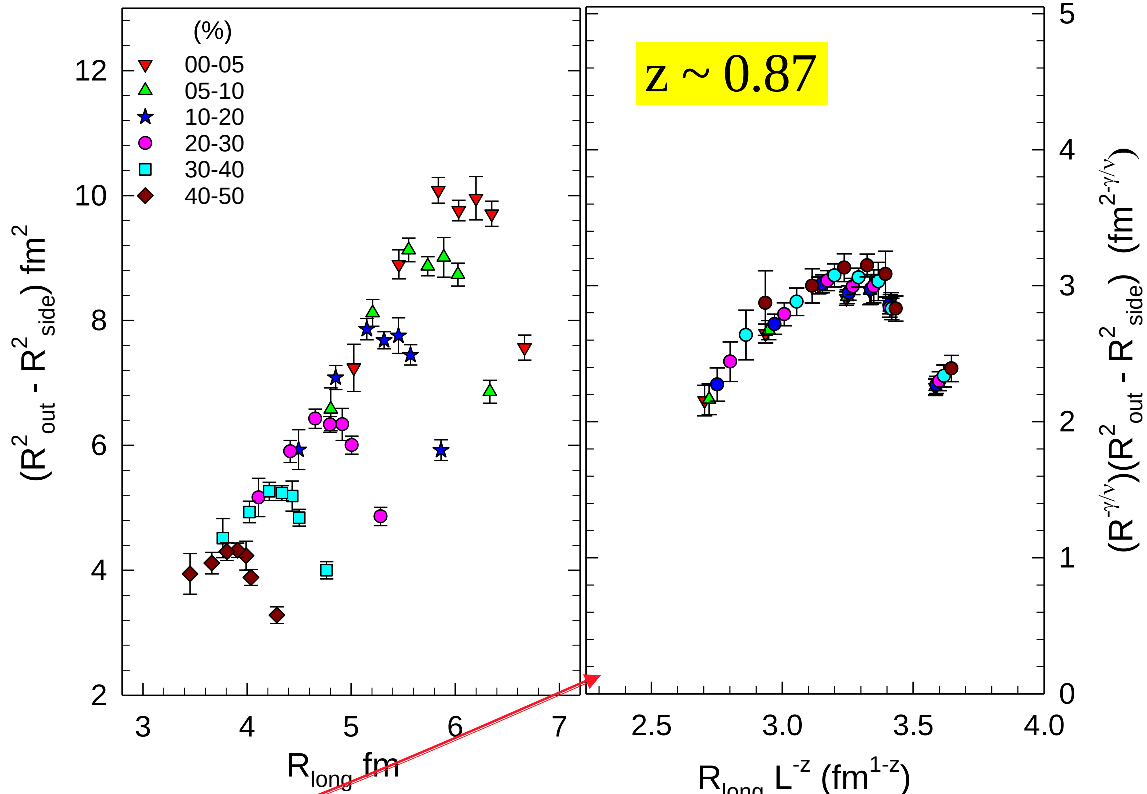
$$\chi(L,T,\tau) = L^{\gamma/\nu} f\left(L^{1/\nu} t_T, \tau L^{-z}\right)$$

*For*
*$T = T_c$*

$$\chi(L,T_c,\tau) = L^{\gamma/\nu} f\left(\tau L^{-z}\right)$$

M. Suzuki,
Prog. Theor. Phys. 58, 1142, 1977

$R_{long} \propto \tau$




**The magnitude of z is similar to the predicted value for $z_s$ but the sign is opposite**

# Epilogue

**Strong experimental indication for the CEP and its location**

<u>(Dynamic) Finite-Size Scalig analysis</u>

- ***3D Ising Model (static) universality class for CEP***
- ***2nd order phase transition***

$T^{cep} \sim 165$ MeV, $\mu_B^{cep} \sim 95$ MeV

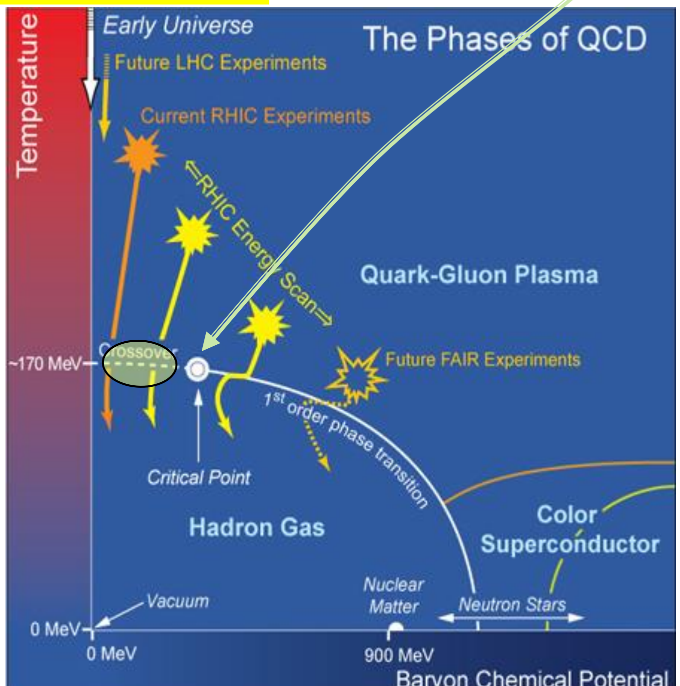
$\nu \sim 0.66$

$\gamma \sim 1.2$

$z \sim 0.87$

**New Data from RHIC (BES-II) together with theoretical modeling, will provide crucial validation tests for the coexistence regions, as well as to firm-up characterization of the CEP!**

✓ Landmark validated
✓ Crossover validated
✓ Deconfinement validated
✓ (Static) Universality class validated
✓ Model H Universality class invalidated?
✓ Other implications!




**Much additional work required to get to "the end of the line"**

*End*

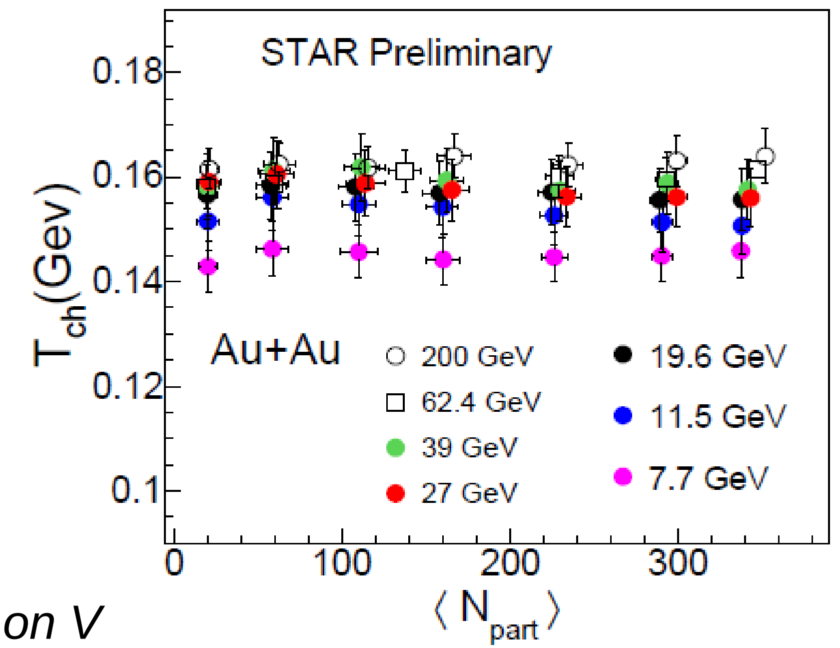
# *Finite – Size Scaling Analysis*

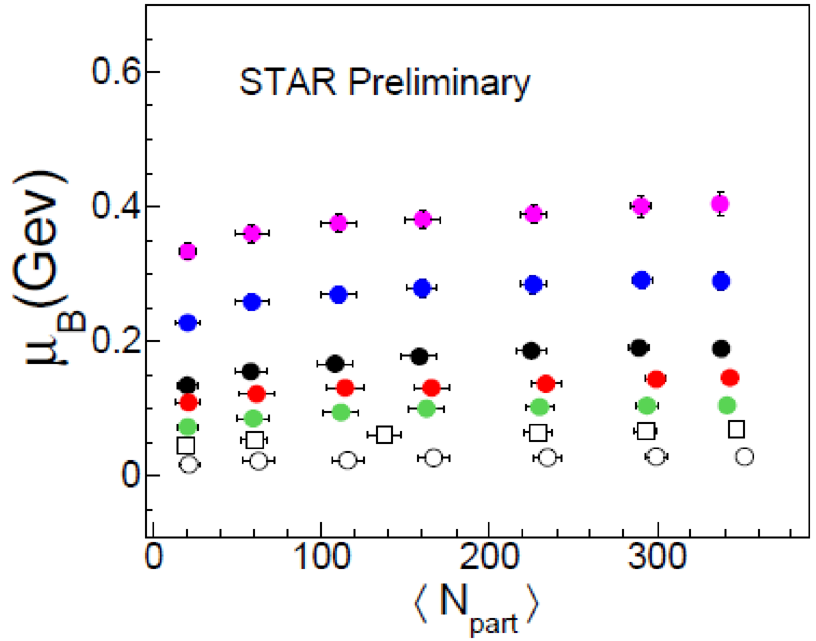
*(only two exponents are independent )*

$$\chi_T^{\max}(V) \sim L^{\gamma/\nu},$$
$$\delta T(V) \sim L^{-\frac{1}{\nu}},$$
$$\tau_T(V) \sim T^{\text{cep}}(V) - T^{\text{cep}}(\infty) \sim L^{-\frac{1}{\nu}},$$

$$(R_{\text{out}}^2 - R_{\text{side}}^2)^{\max} \propto \bar{R}^{\gamma/\nu},$$
$$\sqrt{s_{NN}}(V) = \sqrt{s_{NN}}(\infty) - k \times \bar{R}^{-\frac{1}{\nu}},$$

*Note that* $\left(\mu_B^f, T^f\right)$ *is not strongly dependent on V*




- ✓ ***Locate*** $(T, \mu_B)$ ***position of deconfinement transition and extract critical exponents***
- ✓ ***Determine Universality Class***
- ✓ ***Determine order of the phase transition to identify CEP***