初期宇宙における物理定数の選択と泡宇宙モデル

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INTRODUCTION

- > この世界を特徴づけている物理定数
- 自然界の物理定数: 光速度、プランク定数
- ・ 電子質量、ウォーク質量
- ・ 四つの力(重力、素粒子力)の結合定数

$$lpha_{E}$$
 = 1/137

- 重力定数や電磁気の微細構造定数
- ・ そもそもこれらの値は定数であるか? $\Delta lpha/lpha < 10^{-8}$
- 定数であったとして、その値は、宇宙初期に決定 されていたのか?

> ダークエネルギーは宇宙定数か

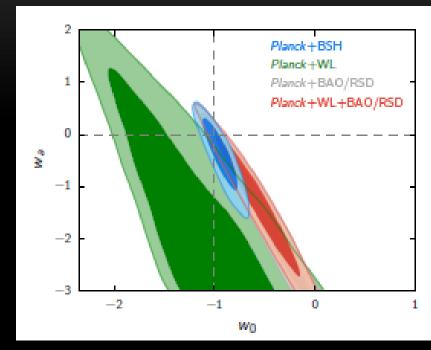
ダークエネルギーの時間変化

$$w(a) = w_0 + (1-a)w_a$$

- ▶ 時間変化⇒ 修正重力理論、 拡張した物質場の導入
- ▶ 定数値⇒ 宇宙項の三つの問題点

1. Smallness

$$\rho_{\Lambda} = \frac{\Lambda m_{\rm pl}^2}{8\pi} \approx 10^{-47} \, {\rm GeV^4} \sim (10^{-3} {\rm eV})^4$$



$$ho_{\Lambda}/
ho_{
m r} \simeq 10^{-55}$$

@electroweak

2. Coincidence

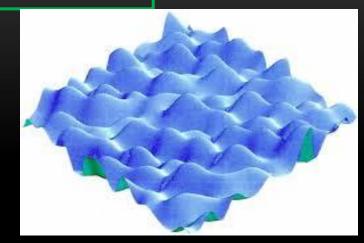
$$O(\rho_{\Lambda}^{(0)}) \sim O(\rho_{\mathrm{DM}}^{(0)})$$

(3.dS) String cosmology favor AdS?

◆ 人間原理、String Landscapeとの関連

10¹⁰⁰ Vacuum with AdS or dS constant

●様々な真空期待値を取ることで、それぞれの真空が<u>様々な値の宇宙項</u>を選択する



例えば、宇宙項が大きいと、



加速膨張が強すぎて最初から空っぽの宇宙のまま膨張し続ける

例えば、宇宙項が小さいと、



誕生後、数百億年以上経って から効き始めるゆっくりした加 速膨張

◆ Coincidence問題への一つのアプローチ

宇宙の構造ができる前に加速膨張が始まっては困るので、 物質密度と宇宙項ほぼ近いオーダーである可能性

INTRODUCTION

- > 真空の相転移と宇宙項の選択
- ダークエネルギー:宇宙定数
- ・ 自然界の物理定数(重力定数や 電磁気の微細構造定数など)



> 異なる定数を取る他宇宙は存在するのが?

Qs aw

していまりませる物理定数の宇宙が存在しているなら、その効果が見えるモデルを作りたい。

Bubble Universe

◆ 泡宇宙 真空の相転移により、 異なる宇宙項を持ついくつかの泡 宇宙が親宇宙の中に生成。 インフレーション提唱時 の初期モデルはこのよう な描像であった。Old inflation(佐藤、Guth 80)

例えば重力定数が異なる泡宇宙はどのようにモデル化できるかっ

$$S = \int d^4x \sqrt{-g} \left[\frac{\Phi^2}{2} R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - V(\Phi) \right],$$

YT and K.Maeda PRD92.023514(2015)

$$G = \frac{G_N}{\langle \Phi \rangle^2}$$

Cosmologica constant

Gravitational constant

$$S = \int d^4x \sqrt{-g} \left[\frac{\Phi^2}{2} R - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \mathcal{V}(\Phi) \right],$$

$V(\Phi)$ Our $\phi_ \phi_ \phi_+$

©Conformal trans

$$rac{ ilde{R}}{2}-rac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi-V(\phi)$$

$$V(\phi) = \frac{\mathcal{V}(\Phi)}{\Phi^4}$$

$$\frac{\mathcal{V}(\Phi)}{\Phi^4} \quad \phi = \phi_0 + \sqrt{7} \ln \Phi$$

$$V(\phi) = \frac{\lambda}{8}(\phi^2 - \mu^2)^2 + \frac{\epsilon}{2\mu}(\phi - \mu) + \Lambda$$

$$\epsilon = \Lambda_{+} - \Lambda_{-}$$

$$\frac{G_{+}}{G_{-}} = \exp\left[-\frac{4\mu}{\sqrt{7}}\right]$$

Nucleation rate

$$ds^2 = a^2(\eta)(-d\eta^2 + dr^2 + r^2d\Omega^2)$$

Flat FRLW = $\mathcal{O}(3)$ symmetry

© Euclidian action

$$\frac{1}{\phi_{-}} \frac{1}{\phi_{+}} \frac{1}{(2\pi(n))^{2}}$$

 $V(\phi)$

$$S = \int d\eta \left[\frac{4\pi}{3} \epsilon a^4(\eta) r^3(\eta) - 4\pi\sigma a^3(\eta) r^2(\eta) \sqrt{1 - (\partial_\eta r(\eta))^2} \right]$$

Tunneling rate

$$\Gamma(\eta_i) \simeq \exp[-2\operatorname{Im} S(\eta_i)]$$

$$ImS = \frac{\pi^2 \epsilon}{3H^4} \sinh^2 \left[\frac{1}{4} \ln(1 + (3H\sigma/\epsilon)^2) \right]$$

$$Im S = \frac{27\pi^2\sigma^4}{4\epsilon^3} \quad \begin{array}{c} H \to 0 \text{ limit} \\ \hline \mathcal{O} \text{ :Tension} \end{array}$$

Wall solution

Control

$$V_0(\phi) = \frac{\lambda}{8} \left(\phi^2 - \mu^2 \right)^2 + \Lambda$$

$$\frac{G_{+}}{G_{-}} = \exp\left[-\frac{4\mu}{\sqrt{7}}\right]$$
 difference of G

$$\partial_r^2 \phi + \frac{2}{r} \partial_r \phi - \frac{dV_0}{d\phi} = 0$$
 :Static EOM

$$\int_{\phi_i}^{\phi} \frac{d\phi}{\sqrt{2(V_0(\phi) - V_0(\phi_i))}} = r - r_i$$

 $d = \frac{(1+b)}{2\mu\sqrt{\lambda}}$

:Thickness

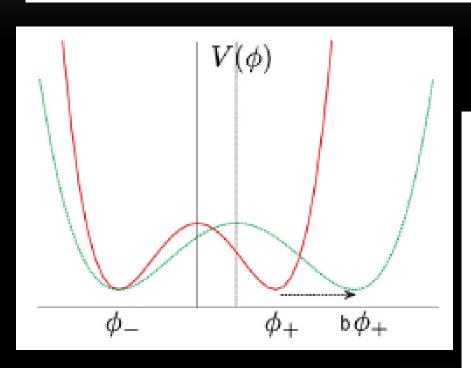
$$\phi(r) = \mu \tanh\left[\frac{\mu\sqrt{\lambda}}{2}(r-r_i)\right]$$
 :Solution

Aim: Compare two different G_+ for same G_-

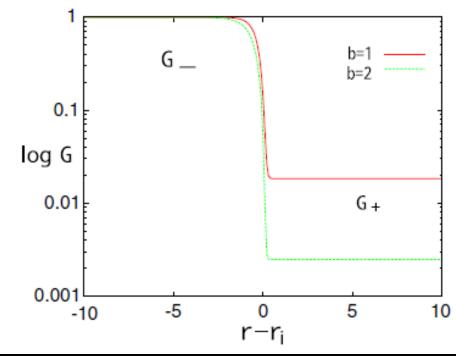
Symmetric potential is not good for that

Shift the potential form with parameter: b

$$V_0(\phi) = \frac{2\lambda}{(1+b)^4} (\phi - b\mu)^2 (\phi + \mu)^2 + \Lambda.$$



$$\frac{G_{+}}{G_{-}} = \exp\left[-\frac{12\sigma}{\sqrt{7\lambda}\mu^{2}}\right]$$



By using wall solution

$$\sigma = \int_0^\infty dr \left[\frac{1}{2} (\partial_r \phi(r))^2 + V(\phi) \right]$$

$$\sigma(b) = \frac{\sqrt{\lambda \mu^3}}{3} (1+b)$$

 $\sigma(b) = \frac{\sqrt{\lambda \mu^3}}{3} (1+b)$ Difference of G+: Tension of wall

Wall Dynamics

$$ds_{\pm}^{2} = -dt_{\pm}^{2} + \exp(2H_{\pm}t_{\pm})(dr_{\pm}^{2} + r_{\pm}^{2}d\Omega^{2}) \qquad H_{+}^{2} - H_{-}^{2} = \epsilon/3$$

Wall Metric

$$ds^{2}|_{\Sigma} = -d\tau^{2} + R^{2}d\Omega^{2} \qquad S_{\mu\nu} = -\sigma h_{\mu\nu}$$

lacktriangle Normal vector to wall n_{μ}

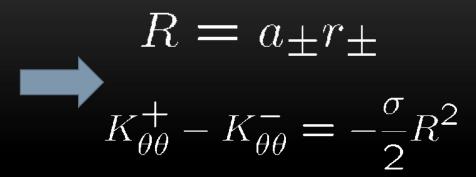
$$n^{\mu}n_{\mu} = 1 \qquad \left(\frac{dt_{\pm}}{d\tau}\right)^{2} - a_{\pm}^{2}\left(\frac{dr_{\pm}}{d\tau}\right)^{2} = 1$$

$$\frac{dt}{d\tau} = \frac{1}{\sqrt{1 - a^{2}(\frac{dr}{dt})^{2}}} \qquad \text{Proper time of wall VS physical time}$$

Junction condition

$$[g_{\mu\nu}]_{-}^{+}|_{\Sigma} = 0,$$

$$[K_{\mu\nu}]_{-}^{+} = -\left(S_{\mu\nu} - \frac{1}{2}h_{\mu\nu}S\right),$$



© EOM for wall dynamics

$$\left\{\frac{dR(\tau)}{d\tau}\right\}^2 = B^2 R^2(\tau) - 1$$

$$R(\tau) = \frac{1}{B} \cosh B \tau$$
.

$$B^2 = H_{\pm}^2 (1 + c_{\pm}^2), \quad c_{\pm}^2 = H_{\pm}^{-2} \left(\frac{\epsilon}{3\sigma} \mp \frac{\sigma}{4} \right)^2$$

$$\ln R = H_{+}t_{+} + \ln r_{+}$$

$$\frac{d(\ln r_{+})}{d\tau} = \frac{\partial_{\tau}(\ln R) \pm H_{+}^{2}R|c_{+}|}{1 - H_{+}^{2}R^{2}}$$

$$r_{+}=rac{r_{0}\cosh B au}{|\sinh B au\pm c_{+}|}$$

Wall dynamics can be solved

$$\frac{-B^2R^2}{H_+}\frac{\partial_t r(t_+)}{r} = 1 - |c_+|\sqrt{B^2R^2 - 1}.$$



Initial radius

$$r_{\pm}(0) = r_{i\pm} = \left| \frac{\epsilon}{3\sigma} \mp \frac{\sigma}{4} \right|^{-1}$$

Asymptotic radius

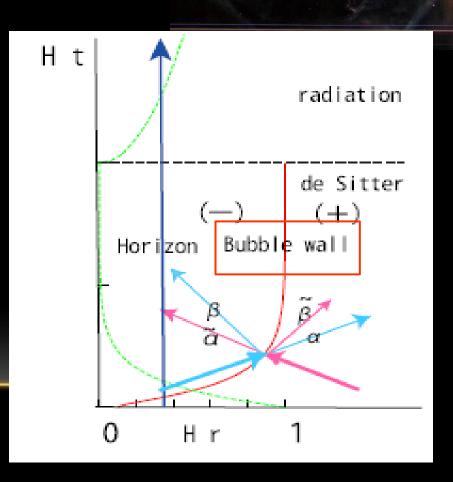
$$r_{\pm}(t\to\infty) = \sqrt{1 + c_{\pm}^2} r_{i\pm}$$

$$r_{\infty\pm} = \sqrt{H_{\pm}^{-2} + (r_{i\pm})^2}$$
.

$$r_{\infty\pm} \sim H_{\pm}^{-1}$$

Comoving radius will approach to be about Hubble size





Cosmological Perturbation

$$ds^{2} = a^{2}[-(1+2\psi)d\eta^{2} + (1-2\psi)(dr^{2} + r^{2}d\Omega^{2})]$$

Slow-roll Inflation

$$a(t) = \exp(Ht), \quad \mathcal{H} = Ha = -\frac{1}{\eta}, \quad -\frac{\dot{\varphi}^2}{2} = \dot{H}$$

• variables $u = \frac{a\psi}{\varphi'}, \quad v = a\left(\delta\varphi + \frac{\varphi'}{H}\psi\right)$

$$u'' - \left(\Delta + \frac{\theta''}{\theta}\right)u = 0$$
 $\theta = \frac{\mathcal{H}}{a\varphi'} = \frac{1}{a\sqrt{2\epsilon_1}}, \quad z = \frac{1}{\theta}$

Slow-roll parameters

$$\frac{\theta''}{\theta} = \frac{1}{\eta^2} \left(\nu^2 - \frac{1}{4} \right), \quad \nu^2 = \frac{1}{4} + \epsilon_1 + \frac{\epsilon_2}{2}$$

$$u = \frac{e^{ikr}}{r} \sqrt{-\eta} \left[C_1 H_{\nu}^{(1)}(-k\eta) + C_2 H_{\nu}^{(2)}(-k\eta) \right]$$

Incoming
wave

$$u_{\rm in} = \frac{e^{ik_-r_-}}{r_-} \frac{\sqrt{-\pi\eta_-}}{2k_-} H_{\nu}^{(1)}(-k_-\eta_-).$$

$$u_{\rm rf} = \frac{e^{-ik_{-}r_{-}}}{r_{-}} \frac{\sqrt{-\pi\eta_{-}}}{2k_{-}} H_{\nu}^{(1)}(-k_{-}\eta_{-}),$$

$$u_{\rm tr} = \frac{e^{ik_+ r_+}}{r_+} \frac{\sqrt{-\pi\eta_+}}{2k_+} H_{\nu}^{(1)}(-k_+\eta_+),$$

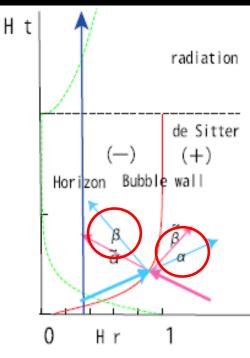


- Reflected wave
- Transmitted wave

Junction condition

$$u_{-}(\tau) = u_{+}(\tau), \quad n_{-}^{\mu} \partial_{\mu} u_{-}(\tau) = n_{+}^{\mu} \partial_{\mu} u_{+}(\tau)$$

$$n_{\pm}^t = \frac{|c_{\pm}| \sinh B\tau - 1}{|\sinh B\tau \pm c_{\pm}|}, \quad n_{\pm}^r = \frac{B^2 r_{0\pm} \cosh B\tau}{H_{\pm}|\sinh B\tau \pm c_{\pm}|^2}$$



derived from solution of wall dynamics

O Analytic Solution @ dS background

$$\beta = \left[-\left(f_{+}(\tau) - f_{-}(\tau) \right) H_{\nu}^{(1)}(-k\eta_{+}) H_{\nu}^{(1)}(-k\eta_{-}) \right. \\ + g_{+}(\tau) H_{\nu+1}^{(1)}(-k\eta_{+}) H_{\nu}^{(1)}(-k\eta_{-}) \\ - g_{-}(\tau) H_{\nu+1}^{(1)}(-k\eta_{-}) H_{\nu}^{(1)}(-k\eta_{+}) \right] \\ \times \left[\left(f_{+}(\tau) - f_{-}(\tau) \right) H_{\nu}^{(1)}(-k\eta_{+}) H_{\nu}^{(2)}(-k\eta_{-}) \right. \\ - g_{+}(\tau) H_{\nu+1}^{(1)}(-k\eta_{+}) H_{\nu}^{(2)}(-k\eta_{-}) \\ + g_{-}(\tau) H_{\nu+1}^{(2)}(-k\eta_{-}) H_{\nu}^{(1)}(-k\eta_{+}) \right]^{-1},$$

$$\nu_{\pm} = \nu = 1/2$$

$$\beta = e^{2ik_{-}r_{-}} \times \frac{f_{+}(\tau) - f_{-}(\tau) + \frac{g_{+}(\tau)}{k_{+}\eta_{+}} - \frac{g_{-}(\tau)}{k_{-}\eta_{-}} + i[g_{+}(\tau) - g_{-}(\tau)]}{-(f_{+}(\tau) - f_{-}^{*}(\tau)) - \frac{g_{+}(\tau)}{k_{+}\eta_{+}} + \frac{g_{-}(\tau)}{k_{-}\eta_{-}} - i[g_{+}(\tau) - g_{-}(\tau)]}$$

 $\epsilon/3 = H_{+}^2 - H_{-}^2 \neq 0$:small

$$\beta = e^{2ik_{-}r_{0}} \times \frac{c_{0}}{\sqrt{1 + c_{0}^{2}}} \frac{\epsilon\ell^{2}}{\cosh(B_{0}\tau)} \left\{ \frac{i}{k_{-}\ell} \left(\sinh(B_{0}\tau) - c_{0}^{-1} \right) + \frac{\sqrt{1 + c_{0}^{2}}}{c_{0}} \cosh(B_{0}\tau) \left(1 + \frac{\eta_{0}}{\ell} \right) - \sinh(B_{0}\tau) \left(1 + \frac{1}{c_{0}^{2}} + \frac{\eta_{0}}{\ell} \right) + \frac{\eta_{0}}{c_{0}\ell} \right\}$$

$$\ell = \sqrt{\frac{3}{\Lambda}} = H_{+}^{-1}, \quad c_0 = \frac{\ell\sigma}{4}, \quad r_0 = \frac{\ell\sqrt{1 + c_0^2}}{c_0},$$

$$B_0 = \frac{\sqrt{1 + c_0^2}}{\ell}, \quad \eta_0 = -\frac{(1 + c_0^2)\ell}{c_0(\sinh(B_0\tau) + c_0)}$$

O Power spectrum

$$\Delta_\zeta^2(k_-) \equiv rac{k_-^3}{2\pi^2} |\zeta_k|^2$$

$$\frac{1}{8\pi^2} \left| (1+ik_-\eta_-) \frac{H_-}{\sqrt{\varepsilon_{1-}}} \beta + \left(1+ik_-\eta_- \frac{\eta_+ \frac{d\eta_-}{d\tau}}{\eta_- \frac{d\eta_+}{d\tau}} \right) \frac{H_+}{\sqrt{\varepsilon_{1+}}} \tilde{\alpha} - \frac{\eta_-}{\frac{d\eta_-}{d\tau}} \frac{H_-}{\sqrt{\varepsilon_{1-}}} \frac{d\beta}{d\tau} - \frac{\eta_+}{\frac{d\eta_+}{d\tau}} \frac{H_+}{\sqrt{\varepsilon_{1+}}} \frac{d\tilde{\alpha}}{d\tau} \right|^2$$

Dimensionless Power spectrum

$$8\pi^2\ell^2\varepsilon_{1+}\Delta_{\zeta}^2(k_{-}) \equiv 1 + s_0 + s_1 + s_2 + \mathcal{O}(\epsilon^2)$$

$$s_{0} = 2\operatorname{Re}\left(\sqrt{\frac{\varepsilon_{1+}}{\varepsilon_{1-}}}\beta + (\tilde{\alpha} - 1)\right),$$

$$s_{1} = 2\operatorname{Re}\left(-\frac{\eta_{-}}{\frac{d\eta_{-}}{d\tau}}\sqrt{\frac{\varepsilon_{1+}}{\varepsilon_{1-}}}\frac{d\beta}{d\tau} - \frac{\eta_{+}}{\frac{d\eta_{+}}{d\tau}}\frac{d\tilde{\alpha}}{d\tau}\right),$$

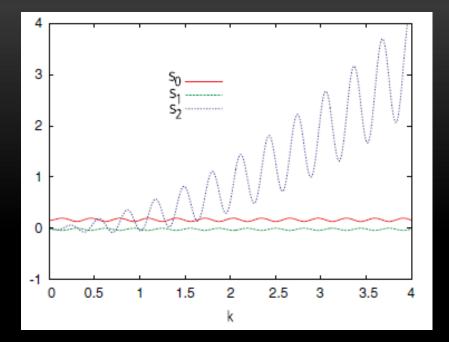
$$s_{2} = 2ik_{-}\eta_{-} \times \operatorname{Im}\left(\sqrt{\frac{\varepsilon_{1+}}{\varepsilon_{1-}}}\beta + \frac{\eta_{+}}{\eta_{-}}\frac{\frac{d\eta_{-}}{d\tau}}{\frac{d\eta_{+}}{d\tau}}(\tilde{\alpha} - 1)\right)$$

$$k_-\eta_-\ll 1$$
 S_1 s_2: Superhorizon

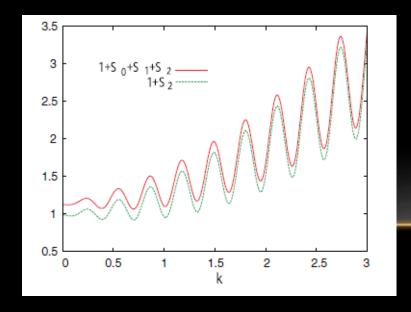
- 反射モード
- 外宇宙からの 透過モード

O Subhorizon scale $k-\eta = \gtrsim 1$

$$s_{2} = \frac{e^{2k_{-}\eta_{0}\sqrt{\varepsilon_{1+}}}\left\{\left(\cos(2k_{-}r_{0}) + \frac{1}{\sqrt{\varepsilon_{1+}}}\right)\frac{c_{0}\sinh(B_{0}\tau) - 1}{k_{-}\ell\sqrt{1 + c_{0}^{2}}\cosh(B_{0}\tau)}\left(\frac{2k_{-}\eta_{0}}{\sqrt{\varepsilon_{1+}}}\right) + \frac{c_{0}\sin(2k_{-}r_{0})}{\sqrt{\varepsilon_{1-}}\sqrt{1 + c_{0}^{2}}\cosh(B_{0}\tau)}\left[\frac{\sqrt{1 + c_{0}^{2}}}{c_{0}}\cosh(B_{0}\tau)\left(1 + \frac{\eta_{0}}{\ell}\right) - \sinh(B_{0}\tau)\left(1 + \frac{1}{c_{0}^{2}} + \frac{\eta_{0}}{\ell}\right) + \frac{\eta_{0}}{c_{0}\ell}\right]\right\}$$



Modulation of Power spectrum



$$m_{
m pl}(=1)$$
 unit $\epsilon=10^{-2}\,\ell^{-2}, ~{
m and}~ \sigma\geq 0.4\,\ell^{-1}$ $\mu\ll m_{
m pl}$

$$\ell = \sqrt{\frac{3}{\Lambda}} = H_+^{-1}$$

S_2 dominant contribution

Increasing Oscillation

$$\sin(2k_{-}r_{0})$$
 $r_{0}=rac{\ell\sqrt{1+c_{0}^{2}}}{c_{0}}$

$$\sigma(b) = \frac{\sqrt{\lambda \mu^3}}{3} (1+b)$$

$$c_0 = \frac{\ell\sigma}{4}$$

$$(k_-\eta_0)^2$$

Most dominant term: Transmitted mode from exterior bubble

Breakdown of perturbative approach

$$\frac{1}{3}\epsilon\ell^2(k_-\eta_0)^2<\mathcal{O}(1)$$

$$\longrightarrow k_{\text{max}} = \sqrt{\frac{3}{\epsilon \ell^2}} \frac{c_0 y_0(\tau)}{(1 + c_0^2)\ell}$$

$$c_0 = 0.1$$
 $k_{\text{max}} \simeq 2.2 \ell^{-3}$

Effect of different Gravitational Constants

$$\phi_{+} = b\mu, \quad \phi_{-}(=\phi_{0}) = -\mu$$

$$G_{+} = e^{-2(1+b)\mu/\sqrt{7}}G_{N}, \quad G_{-} = G_{N}$$

$$\sigma(b) = \frac{\sqrt{\lambda \mu^3}}{3} (1+b)$$

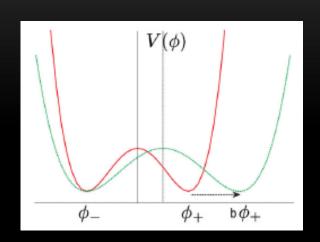
$$b \sim -1$$

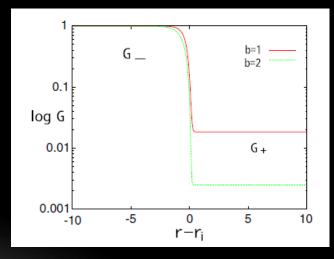
Difference of G is very small

Nucleation rate

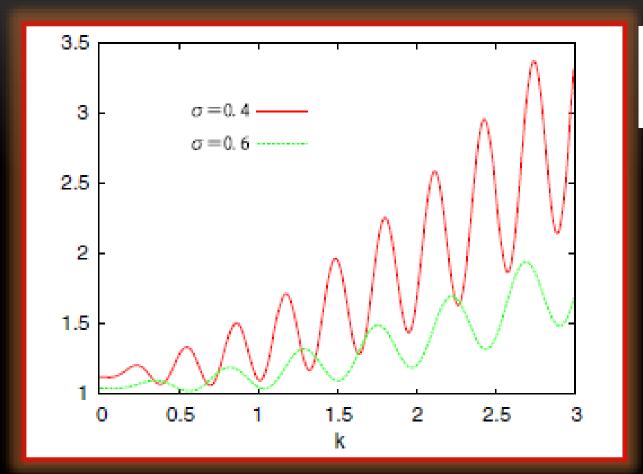
$$\Gamma \simeq \exp\left[-\frac{\pi^2\sigma}{2H^3}\right],$$

Exponential suppressed





 $\sigma \approx 0$ ($b \approx -1$). High nucleation probability



$$\sigma(b) = \frac{\sqrt{\lambda}\mu^3}{3}(1+b),$$

$$\cos(2k - r_0)$$

Oscillation period

$$\Delta k \simeq \pi/r_0 \propto \sigma$$

Period becomes sharper for Smaller σ

When Difference of Both G constants is smaller, Modulation becomes larger

SUMMARY AND DISCUSSION

- ▶ スカラー場の相転移により重力定数の異なる泡宇宙モデルを構築
- ▶ 泡宇宙生成率、泡宇宙の壁の運動を解析し、初期密度揺らぎへの 影響を考察した。
 - ► 揺らぎは我々(内側)の宇宙の揺らぎの波が壁で反射、透過、 さらに外宇宙からの透過波を加えた、 揺らぎの振幅にモードの変更が生じる。
 - ➤ Subhorizonで増大する振動関数がdominantになる。 これらは外宇宙からの影響。
 - ▶ 重力定数差が小さくなる場合に、生成率も大きく、揺らぎへの効果 も大きくなる。振動の周期もよりシャープになる。
 - 外からの透過波の影響が与えるCMB への影響
 - 他の物理定数に対する同様の機構
 - 重力と運動項との結合による泡宇宙
 - □ 泡宇宙の衝突現象